Uncertainty, Wages, and the Business Cycle*

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PRELIMINARY

Abstract

We show that occasional deviations from efficient wage-setting generate strong and state-dependent amplification of exogenous uncertainty shocks and contribute to explain the observed countercyclicality of empirical measures of aggregate uncertainty. Central to our analysis is the existence of matching frictions in the labor market and an occasionally binding constraint on downward wage adjustment. The wage constraint enhances the concavity of the firms' hiring rule, generating an endogenous profit-risk premium. In turn, uncertainty shocks increase the profit-risk premium when the economy operates close to the wage constraint. This implies that higher uncertainty can severely deepen a recession although its impact is weaker on average. Additionally, the variance of the unforecastable component of future economic outcomes always increases at times of low economic activity. Thus, measured uncertainty rises in a recession even in the absence of uncertainty shocks. We obtain our results using a fourth-order Taylor expansion to the model policy functions, approximating the non-convexity in the wage decision rule with a novel implementation of penalty function methods.

JEL Codes: E32, E2, E6.

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1 Introduction

The cyclical behavior of wages and its implications for the propagation of business cycle shocks are central questions in macroeconomics. Since Keynes (1936), a vast literature suggests that wage-setting frictions have important implications for macroeconomic dynamics. For instance, sluggish wage adjustment plays a key role for the transmission of monetary policy shocks in estimated general equilibrium models (Christiano, Eichenbaum, and Evans, 2005) and it accounts for the largest share of cyclical fluctuations in the labor wedge (Galí, Gertler, and López-Salido, 2007). More recently, the slower-than-expected average wage growth experienced by the U.S. economy led some economists to conclude that frictions preventing sufficient downward wage adjustment exacerbated the fall in employment during the Great Recession (see Daly, Hobijn, and Lucking, 2012, Shimer, 2012, and Yellen, 2014).

The role of sluggish wage adjustment for the transmission of demand and supply shocks is well understood. Less is known about the consequences of wage-setting frictions at times of high macroeconomic uncertainty, a potential factor shaping the depth and duration of recessions as suggested by recent research.[1] This paper shows that occasional deviations from efficient wage-setting generate strong and state-dependent amplification of exogenous uncertainty shocks and contribute to explain the observed countercyclicality of empirical measures of aggregate uncertainty. Central to our analysis is the existence of an occasionally binding constraint (OBC) on downward wage adjustment.

We cast the analysis in the context of a general equilibrium model featuring search and matching frictions in the labor market, and introduce a lower bound for real wages at the bargaining stage. The constraint is binding at times of low aggregate demand, implying that wage cuts are feasible in a recession provided that the initial wage level is sufficiently high. By contrast, in a prolonged or severe recession, downward wage adjustment is precluded. When the constraint is not binding, the wage payment splits the match surplus according to efficient Nash bargaining. Our assumptions formalize the empirical observation that while firms face constraints in their ability to reduce hourly wages during an economic downturn, there is flexibility in adjusting workers' total compensation by curbing performance pay, bonuses, and other profit-sharing arrangements—an empirical finding documented by a recent and growing literature discussed below.

[1] Heightened uncertainty has been suggested as a major contributor to the magnitude of the slump experienced by the U.S. economy over the period 2007-2012, including the dramatic increase in the spell of unemployment duration, the historically low vacancy yield, and the fall in recruiting intensity. Stock and Watson (2012) and Baker, Bloom, and Davis (2012) estimate that the increase in uncertainty explains a substantial portion in the fall of U.S. GDP during the Great Recession. Leduc and Liu (2012) find that increased policy uncertainty accounts for two-thirds of the shifts of the Beveridge curve over the same period.

Our analysis yields two main results. First, occasional deviations from efficient wage-setting generate strong and state-dependent amplification of exogenous fluctuations in the variance of future productivity shocks: An increase in uncertainty has substantially stronger recessionary effects when first-moment shocks have already increased unemployment sufficiently above its average level. Thus, higher uncertainty can severely deepen a recession although its impact is weaker on average. Using a standard parameterization, we find that when productivity is 1 percent below trend, a two-standard deviations increase in the volatility of productivity results in an additional fall in employment of about 0.5 percentage points. By contrast, both flexible wage setting and wage rigidity that binds at all times imply a negligible propagation of exogenous uncertainty shocks, even if these two wage-setting protocols result in very different business cycle responses to first-moment shocks.

Second, the existence of an occasionally binding constraint (OBC) on wage negotiations implies that the variance of the unforecastable component of future economic outcomes—a well-documented countercyclical measure of uncertainty—always increases at times of low economic activity.[2] Thus, measured uncertainty increases during recessions even in the absence of exogenous uncertainty shocks.

Our results offer a novel perspective about the link between uncertainty shocks and the countercyclicality of several empirical measures of economic uncertainty documented in post-war U.S. data. In benchmark business cycle models, where time-varying uncertainty is the result of stochastic volatility of exogenous shocks, the impact of uncertainty shocks is independent of the state of the business cycle. As a consequence, when estimated, these models explain the counterciclicality of measured uncertainty in one of two ways. Either uncertainty shocks explain a large share of the variance of output or, when the variance of output is mostly driven by first-moment shocks, uncertainty shocks are ex-post negatively correlated with first-moment shocks (Bloom, 2009, Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe, 2011, Christiano, Motto, and Rostagno, 2014). Our paper shows that neither explanation is necessarily true. On the one hand, countercyclical measured uncertainty can stem from uncertainty shocks even if those shocks are on average acyclical, since the covariance between output and exogenous uncertainty is higher in a recession relative to an expansion. On the other hand, part of the variation in measured uncertainty reflects the endogenous response of the economy to first-moment shocks, akin to models in which time-varying uncertainty is an equilibrium outcome resulting from agents' optimal decisions as in Bachmann and Moscarini (2011), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014), and Saijo (2012).

[2] The aggregate measures of uncertainty considered in the empirical literature include the volatility of stock and bond markets, the volatility of output and exchange rates, as well as measures of disagreement among professional forecasters, and their self-reported subjective forecast uncertainty (Bloom, 2014).

Our findings also suggest that the identification of uncertainty shocks in reduced-form econometric models such as VARs should account for the state-dependence of their impact.[3] From a policy perspective, our results have two implications. First, the optimal response of policymakers to changes in exogenous uncertainty depends on the state of the business cycle. Second, inefficient markup fluctuations and nominal rigidities need not to be central to the propagation of uncertainty shocks. This result typically eludes benchmark business cycle models in which fluctuations in uncertainty that are consistent with the U.S. data generate very modest aggregate effects except under strong assumptions about nominal rigidity (Bachmann and Bayer, 2011, Cesa-Bianchi and Fernandez-Corugedo, 2014, Chugh, 2013, and Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez, 2011). Moreover, in our model, wages can deviate from their efficient level yet remaining incentive-compatible at all times. By contrast, as noted by Barro (1977), rigid wages are not renegotiation-proof in Walrasian labor markets.

The intuition for our results is the following. Labor market frictions imply that job creation depends on the expected present discounted value of the stream of profits generated by a match over its tenure. Since agents enter into multi-period employment contracts, the constraint on wage bargaining influences job creation by affecting the present discounted value of the stream of wage payments. As we illustrate in a simple three-period model, the OBC enhances the concavity of firm profits with respect to productivity, generating a large profit risk-premium. The closer the economy operates to the constraint, the larger the difference between the expected stream of profits and the profit stream from the expected path of productivity. At times of low aggregate demand and employment, an increase in the probability of more extreme productivity realizations leads to a sizable increase in the profit-risk premium, since the wage constraint is expected to bind with higher probability in the future. In turn, the sharp reduction in the firm's expected surplus leads to an immediate reduction in job creation. By contrast, during economic expansions, firms operate far away from the wage constraint. Accordingly, an increase in uncertainty does not result in a large change in the expected surplus from job creation.

When wages are unconstrained or unconditionally rigid, or when the constraint is never binding for the wages of new hires, the effect of uncertainty shocks becomes negligible. This result obtains because eliminating the OBC leaves little nonlinearity in the model for risk consideration to have a substantial impact on agents' optimal choices through the wage channel. Thus, the impact of uncertainty shocks

[3]Several authors have documented the state-dependent effect of first-moment shocks. For instance, Hubrich and Tetlow (2012) show that financial shocks are more recessionary at times of high financial stress. Caggiano, Castelnuovo, and Groshenny (2013) address the state-dependent impact of uncertainty shocks in the U.S. business cycle by estimating a smooth-transition VAR.

does not depend on the size of the inefficiency in wage setting, but on the implied degree of concavity in the profit function.

Turning to the behavior of measured uncertainty, our model implies that the forecast error variance of output and employment displays pronounced countercyclical movements even in response to first-moment shocks, i.e., absent exogenous fluctuations in uncertainty. The reason is that agents anticipate that job creation responds more strongly to productivity innovations when the economy operates close to the wage constraint. By contrast, when the constraint binds with low probability, efficient surplus-splitting results in cyclical wages, lowering the volatility of output and employment for any given realization of productivity. As a result, the forecast error variance becomes less sensitive to aggregate conditions.

An extensive empirical literature documents the existence of constraints on downward wage adjustment for a large number of countries (see, for instance, the cross-country evidence in Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward, 2007). Such constraints reflect both institutional and non-institutional factors. On one side, in countries where collective bargaining agreements are the predominant wage contract (e.g. European economies) there is a *de jure* lower bound for wages.[4] On the other, as shown by the literature on efficiency wages, firms may find it unprofitable to lower wages even in the absence of institutional constraints preventing wage adjustment. For instance, as argued by Akerlof and Yellen (1990), workers respond to a wage-cut below the perceived fair wage by lowering effort, leaving little incentives for firms to cut pay in the first place.[5]

Our assumption that downward wage rigidity binds only occasionally is consistent with recent evidence showing that firms have alternative margins to reduce total workers' compensation, even when the base hourly wage is rigid. The International Labour Office (2010) documents that collective bargaining agreements routinely link workers' compensation to performance, a worldwide trend not limited to European economies. Moreover, since the beginning of the 1990s, a large number of European countries began to adopt two-tier bargaining structures in which plant-level bargaining supplements national or industry-wide (multi-employer) agreements, taking the pay agreement established at the multi-employer level as a floor—in plant-level negotiations, wages can be adjusted only above the wage floor established at the national or industry level (Boeri, 2015). Boeri (2014), using data

[4] The coverage rate is above 70 percent in Austria, Belgium, Denmark, Finland, France, Greece, Italy, Netherlands, Portugal, Romania, Slovenia, Spain, Sweden, and between 50 and 70 percent in Germany (International Labour Office, 2008). In the U.S., where the importance of collective bargaining agreements has declined to 15 percent, Clemens and Wither (2014) estimate that the increase in the average effective minimum wage over the late 2000s accounts for 14 percent of the total decline in the employment-population ratio over the same period in the U.S.

[5] Survey evidence from interviews with managers shows that employers are reluctant to cut wages of new hires believing that the pay-scale difference with incumbents would hurt morale (Bewley, 1995).

for thirteen European countries, shows that the coverage of plant-level bargaining agreements in firms with more than two-hundred employees was above 50 percent over the period 2007-2009. Consistent with this evidence, Carneiro, Guimar, and Portugal (2012) and Jung and Schnabel (2011) show that in Portugal and Germany firms adopt a "wage cushion"—a premium over the contractual wage agreed upon in the collective bargaining agreement—which is highly cyclical for new hires, allowing firms to lower wages up to the base pay level prescribed by the contractual wage.

Using data from the Panel Study of Income Dynamics, Lemieux and Farés (2001) and Lemieux, MacLeod, and Parent (2009) find that, since the 1980s, the incidence of performance-pay contracts (performance bonuses, commissions, and piece-rate contracts) rose to affect about 45 percent of jobs in the U.S., with a median share of performance pay in total earnings equal to 4 percent. Champagne and Kurmann (2013), using U.S. longitudinal worker-employer administrative data, document an even larger incidence of performance-pay contracts, which rose from 30 to 60 percent since the 1980s. Bryson, Freeman, Lucifora, Pellizzari, and Perotin (2012) report that in the 2006 U.S. General Social Survey, the share of workers receiving pay in the form of profit or gains-sharing was about 40 percent, while the share of workers receiving some form of incentive pay exceeded 50 percent. Also in sectors that remained unionized, automatic wage adjustments linked to productivity that featured U.S. collective wage agreements in the post-war period have evolved into performance bonuses.[6]

Partial downward adjustments of worker's compensation also reflects the ability of firms to reduce the number of hours worked while demanding increased effort.[7] For instance, Lemieux, MacLeod, and Parent (2009) find that jobs not covered by performance-pay scheme experience higher hours volatility. Similarly, Kurmann and Spletzer (2014) show that while downward wage rigidity is indeed a constraint for the adjustment of the base hourly wage, firms react to this constraint by modifying hours worked, together with overtime pay and performance bonuses.[8]

[6]For example, in 1948 the United Auto Workers (UAW) agreement with General Motors provided an annual real wage increases of 2.5 percent to account for productivity increases. The 2015 proposed agreement between UAW and Fiat Chrisler includes bonuses up to $13,000 conditional on productivity targets being met (*The New York Times*, page B1, October 1, 2015).

[7]Bils, Chang, and Kim (2014) build a search and matching model where firms can bargain over worker-effort even if wages cannot be adjusted.

[8]Hours adjustment can only ensure limited downward flexibility, since workers cannot vary the intensive margin at will, but effectively choose among bundles of wages and hours of work (Rogerson, 2011). Firms may require employees to work a fixed number of hours because of technological constraints, set-up costs, supervisory time and coordination issues. This is consistent with the observation that for many activities firms do not consider part-time workers (Prescott, Rogerson, and Wallenius, 2009). The fact that workers choose among "packages" of total compensation and hours of work was already exploited by Bernanke (1986) to explain the puzzling increase in real hourly earnings during the Great Depression in several manufacturing sectors. Bernanke observed that while firms used short work weeks to reduce the labor input, firms could not cut weekly earnings below a minimum level ensuring subsistence to the workers. Thus the hourly wage rose to compensate for the fall in weekly hours.

Finally, recent research documents that the measure of labor cost relevant to explain employment growth—the wage of newly hired workers—is more cyclical relative to the wage of incumbents (Haefke, Sonntag, and van Rens, 2013, Martins, Solon, and Thomas, 2012, Pissarides, 2009), although Bils, Chang, and Kim (2014) present evidence consistent with the notion that entry wages are partly determined by the wages of incumbents.

The recent evidence about the ability of firms to partially adjust workers' compensation contributes to explain the weak average impact of downward wage rigidity on aggregate labor market outcomes (Elsby, 2009 and Lemieux and Farés, 2001). It also motivates our departure from the traditional approach in the literature which assumes that downward wage rigidity binds at all times, such that wage cuts can never be implemented. We parametrize the OBC to match existing evidence about the fraction of real wage cuts prevented by downward rigidity. The ergodic wage distribution implies that the wage constraint does not bind in 85 percent of wage outcomes.

Our paper provides a methodological contribution by solving a model with occasionally binding constraints and stochastic volatility using a novel implementation of the penalty function method. Such method, a well established approach in the field of applied mathematics, transforms an optimization problem containing an inequality constraint into an unconstrained problem. Since the penalty term in the unconstrained problem is assumed to be a smooth function of the variables appearing in the original OBC, it is possible to use perturbation techniques to approximate the model solution to an arbitrary degree of accuracy, without relying on global solution methods.

The innovation of our approach consists in assuming that the penalty function is a fourth-order polynomial whose coefficients are chosen to minimize squared deviations from a grid of values defined over the region of the state space of economic interest. Effectively, the equilibrium polynomial law of motion for the penalty function is obtained through a global approximation method, while the policy functions for the model are derived using a local approximation which takes the polynomial penalty function as given. Relative to standard fourth-order Taylor approximations of continuos and twice-differentiable penalty functions, our approach improves the accuracy of the approximation in the portion of the state space that is relevant for the analysis of aggregate fluctuations in standard business cycle models. In so doing we address important limitations of Taylor expansions of penalty functions highlighted by Den Haan and De Wind (2012).

The paper is organized as follows. Section 2 discusses the related literature. Section 3 illustrates analytically the relationship between increased uncertainty and job creation in a partial-equilibrium, search and matching model with an OBC on wage bargaining. Section 4 presents the general equilibrium model. Section 5 discusses the parameterization of the model, while Section 6 illustrates the solution method. Section 7 studies the effects of uncertainty shocks and the behavior of measured uncertainty over the business cycle. Section 8 concludes.

2 Related Literature

In the recent past, various contributions addressed the role of downward wage rigidity for the propagation of first-moment shocks and the conduct of macroeconomic policy. Schmitt-Grohé and Uribe (2013) and Schmitt-Grohé and Uribe (2015) study the consequences of downward nominal wage rigidity for exchange rate policy. They build an open economy model featuring an occasionally binding constraint on downward wage adjustment and involuntary unemployment, in a framework where search frictions play no role. Abbritti and Fahr (2013), Kim and Ruge-Murcia (2009), and Kim and Ruge-Murcia (2011) study the role of asymmetric wage adjustment costs for the conduct of monetary policy. In contrast to these studies, we focus on the role of occasional deviations from efficient wage-setting for the macroeconomic consequences of aggregate uncertainty.

The relationship between macroeconomic uncertainty and business cycle dynamics is the subject of a large and growing literature. Part of this research focuses on the transmission of exogenous uncertainty shocks. One strand considers the role of fixed costs and investment irreversibility. For instance, Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) show that both aggregate and firm-level volatility shocks can delay investment decisions, resulting in a contraction of output.[9] Schaal (2012) exploits a similar mechanism in a model that features a fixed cost of hiring, firm-worker random matching, and endogenous firm entry and exit. In our model, inaction does not play any role. The reason is that in the symmetric equilibrium of the search and matching model, the option value of waiting to create a new job is zero, despite the presence of sunk vacancy costs and job separation risk.

A second strand of the literature focuses on endogenous markup fluctuations and nominal rigidities. Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2011) show that, because of nominal rigidities, prices cannot fully accommodate the drop in demand triggered by precautionary behavior in response to uncertainty shocks. When volatility shocks make future marginal costs and demand harder to predict, firms have an incentive to bias their prices upward with respect to the level they would otherwise pick in order to maximize expected profits, lowering demand and equilibrium

[9] Ambiguity aversion as in Ilut and Schneider (2014) provides an alternative avenue to explain the impact on the economy of uncertainty.

output. Basu and Bundick (2012) show that in the standard neoclassical growth model, uncertainty shocks are not capable of producing business-cycle comovements among key macroeconomic variables, including output and hours. Countercyclical markups resulting from sticky prices allow uncertainty shocks to generate fluctuations consistent with the empirical comovements observed in business cycles. Leduc and Liu (2012) consider a standard search and matching model with sticky prices and real wage rigidity. Similarly to the results of much of the literature, in their framework uncertainty shocks have a non-negligible impact on output and employment only when nominal price rigidities are strong. By contrast, in the absence of fluctuations in price-markups, the impact of uncertainty on employment is reduced by a factor of ten. Finally, an active strand of the literature studies the transmission of uncertainty shocks when the economy is at the zero lower bound on nominal interest rates. These models rely on nominal rigidities to induce real effects of monetary policy. In a benchmark New Keynesian model, Nakata (2013) finds that an increase in the variance of shocks to the discount factor is significantly more recessionary when the economy is at the zero lower bound relative to normal times. Plante, Richter, and Throckmorton (2014) show that the output forecast error variance increases endogenously when the economy is at or close to the zero-lower bound.

In our framework, time-varying markups and nominal rigidities play no role for the transmission of uncertainty shocks. Furthermore, in Nakata (2013) and Plante, Richter, and Throckmorton (2014) monetary policy is captured by an exogenous truncated interest rule. Thus, the monetary authority does not account for the existence of an occasionally binding constraint in its decision-making. By contrast, in our setup agents account for the occasionally binding constraint on wages when negotiating the wage contract.

A third strand of the literature focuses on the role of financial frictions for the propagation of firm-level volatility shocks. Arellano, Bai, and Kehoe (2012) consider a model where producers face a credit constraint that affects their ability to finance the cost of labor. When the variance of the idiosyncratic shocks increases, the probability of default increases at a given level of employment. As a result, firms become more cautious and decrease employment, leading to a fall in aggregate output. In a related framework, Gilchrist, Sim, and Zakrajsek (2014) discuss the role of limited liability for the impact of changes in uncertainty on firms' investment decision, showing that agency problems in financial intermediations are central to the transmission of uncertainty shocks. Christiano, Motto, and Rostagno (2014) show that volatility shocks to the quality of capital account for a significant portion of output fluctuations in a New-Keynesian model with a Bernanke-Gertler-Gilchrist financial accelerator mechanism. While our paper focuses on fluctuations in aggregate uncertainty, thus abstracting from cross-sectional variation in firm productivity, our analysis complements the perspective of these studies.

Our results are also related to a recent literature that models uncertainty as an endogenous outcome of recessions. Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014) and Saijo (2012) propose models where the information flow depends on aggregate variables. Since signals become less informative during recessions, uncertainty increases endogenously, further depressing economic activity. Saijo (2012) shows that nominal rigidities remain essential to generate non-negligible aggregate effects also in a model where time-varying uncertainty is endogenous. In our model, there is no feedback mechanism from the choice of agents to the level of uncertainty in the economy, even if it will appear to an outside observer that agents' dispersion of forecast increases in recessions. Thus, in contrast to information-inference based models of endogenous uncertainty, increasing the flow of information across agents has no impact on the severity or length of recessions.

Finally, our paper is related to Ilut, Kehrig, and Schneider (2014), who document negative skewness of employment growth in both the cross section and the time series of establishment-level U.S. Census data. They also show that the response of employment growth to TFP shocks is asymmetric, with net employment losses responding more strongly to productivity shocks than net employment gains. They rationalize these findings with a model in which firms adjust asymmetrically to dispersed but correlated signals about productivity. While their focus is on the role of firm aversion to Knightian uncertainty (ambiguity), we study the relationship between wage-setting and hiring frictions as the key source of concavity in firms' responses to second-moment shocks.

3 A Simple Model: The Impact of Uncertainty Shocks with Occasionally Binding Constraints on Wage Bargaining

This section illustrates how the existence of an occasionally binding constraint on wage bargaining affects the propagation of uncertainty shocks in a partial-equilibrium version of the standard search and matching model.[10] The purpose is to build intuition for the results presented in the next sections in the context of a general equilibrium model. We provide all the mathematical details for the results and propositions presented below in the Appendix.

The mechanism we highlight is the following. Search and matching frictions in the labor market imply that current job creation depends on the expected, present discounted value of the stream of profits generated by a productive match over its tenure. Since firms decide job creation based on the

[10] In a similar framework, Cabrales and Hopenhayn (1997) discuss the impact of downward wage-growth rigidity on job creation.

expected profit corresponding to future productivity realizations, the concavity in the firm's profit function generated by the constraint on wage bargaining results in a profit-risk premium. In turn, higher dispersion of future productivity realizations increases the profit risk premium, inducing firms to optimally reduce current hiring and employment.

Consider the following partial equilibrium model. In order to fill a vacant job, a representative firm needs to post vacancies, incurring a cost κ. With probability q_t the vacancy is filled, and the job remains productive until an exogenous separation shock occurs. The value of a filled vacancy is the difference between the expected present discounted value of the stream of revenues generated by the match over the course of its tenure, $\overline{y}_t$, and the present discounted value of the stream of wage payments accruing to the worker, $\overline{w}_t$. In equilibrium, free entry implies that the value of a filled vacancy is equated to its expected filling cost:

$$\overline{y}_t - \overline{w}_t = \frac{\kappa}{q_t}. \tag{1}$$

We now make a number of simplifying assumptions. First, matches have a fixed tenure of two periods and production occurs at time $t = 1, 2$. Second, we abstract from discounting. Third, we assume that new matches are formed at the end of period $t = 0$, so that firms choose the number of vacancies to be posted before observing the realization of productivity. Finally, without loss of generality, we set the flow value of unemployment equal to zero.

A firm-worker pair produces output $y_t \equiv y(Z_t)$, where Z_t is a random productivity draw. Let w_t be the wage payment accruing to the worker. Our assumptions imply that, at the time of the hiring decision, the expected stream of revenues and wage payments are, respectively:

$$\overline{y}_1 = E_0 \sum_{t=1}^{2} y_t, \;\; \overline{w}_1 = E_0 \sum_{t=1}^{2} w_t, \tag{2}$$

where the price of output is normalized to one. Wage-setting in period 1 and 2 splits the surplus of the match between the firm and the worker according to a predetermined function of the productivity realization.

We study the consequences of uncertainty for job creation under three alternative wage-setting rules: (i) fixed wages; (ii) unconstrained Nash bargaining, and (iii) constrained Nash bargaining. Under fixed wages, $w_1^{fixed} = w_2^{fixed} = w$, where w is any feasible value in the bargaining set. When wages are not fixed, Nash bargaining sets a wage schedule conditional on the period-t productivity realization. Under unconstrained Nash bargaining, the time-t wage solves:

$$w_t^{nash} = \arg\max J_t^{1-\eta} W_t^{\eta},$$

where J_t and W_t are, respectively, the value of the match to the firm and the worker conditional on time-t information, and $\eta \in (0,1)$ is the worker's bargaining power. Under constrained Nash-bargaining, the wage cannot fall below w_m, such that

$$\widetilde{w}_t^{nash} = \arg\max_{w_t \geq w_m} J_t^{1-\eta} W_t^{\eta}.$$

For a given wage protocol, we solve the model by backward induction. In the second period, the arguments of the Nash bargaining problem J_2 and W_2 only depend on Z_2, since the continuation value of the match is zero. For this reason, $J_2 = y(Z_2) - w_2$ corresponds to the second-period profit, while $W_2 = w_2$ is the second-period wage. By contrast, in the first period, the values of the match to the firm and the worker depend on Z_1 as well as on the expected flow values at time $t = 2$:

$$J_1 = y(Z_t) - w_1 + E_1(J_2),$$
$$W_1 = w_1 + E_1(W_2).$$

Job creation depends on the expected value of the stream of profits in periods 1 and 2, taking as given the contract renegotiation conditional on Z_t. To simplify the exposition, from now on we set $\eta = 0.5$ and assume $y_t = 2Z_t$, where the c.d.f. $F(Z_t)$ is uniform with support $[Z^a, Z^b] \in \mathbb{R}^+$.

We now discuss the effects of an exogenous increase in uncertainty about future productivity. First, we show under what conditions there exists a profit risk-premium for period-2 profits under constrained Nash bargaining. Next, we show that the profit risk-premium in period 2 affects the total expected value of the match, $(\overline{y}_1 - \overline{w}_1)$.

Proposition 1 *Under unconstrained Nash-bargaining or fully rigid wages, an increase at $t = 2$ in the variance of productivity Z for given $E(Z_2)$ does not affect period 2 expected profits. Expected profits fall when wage bargaining is constrained by the lower bound w_m.*

With unconstrained Nash bargaining, second-period wages and expected profits are given by

$$w_2^{nash} = \frac{y_2(Z_2)}{2}, \tag{3}$$

$$E_1(J_2) \equiv \int\limits_{Z \in [Z^a, Z^b]} \frac{y_2(Z_2)}{2} dF(Z_2) = E(Z_2), \tag{4}$$

Given our assumptions, profits are a linear function of productivity Z_2. Therefore, expected profits $E_1(J_2)$ are unaffected by an increase in the variance of Z_2.

It is a well-established results that in the baseline search and matching model, Nash bargaining implies a low elasticity of profits to productivity. Shimer (2005) notes that this result ultimately explains the inability of the model to reproduce the observed volatility of vacancies and employment relative to output. In the literature about the "Shimer puzzle," wage rigidity has been shown to amplify the impact of productivity shocks on profits and job creation. This result, however, does not extend to second-moment shocks, even when wages are fully rigid. In our partial-equilibrium model, setting the wage equal to any incentive-compatible constant value results in a profit function J_2 with a higher first derivative with respect to productivity, but still linear in productivity. Thus, as for the case of unconstrained Nash bargaining, completely rigid wages also prevent changes in uncertainty from having any impact on $E_1(J_2)$.

Under constrained Nash bargaining, $\tilde{w}_2^{nash} = \max[w_2^{nash}, w_m]$. Define Z^m as the productivity threshold such that $w_2^{nash} = w_m$ in (3). Expected profits $E_1(J_2)$ can then be written as follows:

$$E_1(J_2) = \int\limits_{Z^a}^{Z^m} [y_2(Z_2) - w_m]\, dF(Z) + \int\limits_{Z^m}^{Z^b} \frac{y_2(Z_2)}{2} dF(Z). \tag{5}$$

An increase in the variance of Z_2 increases the range over which the two integrals are calculated, and rescales the density function $dF(Z)$. As a consequence, the relative weight of the productivity realizations across the two integrals will change, since Z^a and Z^b take different values, while Z^m is not affected by the variance increase. This will in turn affect the expected profit, given that the firm's surplus share is constant over $[Z^m, Z^b]$ but decreasing in Z from Z^m to Z^a. We show in the Appendix that, as a consequence, higher uncertainty about Z_2 always decreases the expected profit $E_1(J_2)$, and that the fall in profits is larger the higher the variance of Z. These results depend on two features of the model. First, the distribution of Z_2 with a lower variance second-order stochastically dominates the distribution of Z_2 with a higher variance. Second, the wage constraint results in a concave profit function J_2.

If the profit function is not concave over the whole domain, an increase in variance may lead to higher profits. To discuss this point, it is instructive to examine the limiting case where the first integral in the profit equation (5) is equal to zero for $Z < Z^m$, assuming that workers receive all the surplus. In the Appendix, we show that under these alternative assumptions, a marginal increase in the dispersion measure σ - equal to the range of Z - lowers $E_1(J_2)$ provided that Z^m, w_m, Z^a, and Z^b are such that:

$$\left\{ [E(Z)]^2 - Z^{m^2} \right\} > \frac{\sigma^2}{4}. \tag{6}$$

Equation (6) implies that $E_1(J_2)$ falls only if the productivity cutoff Z^m is sufficiently below the unconditional productivity average value, $E(Z_2)$. In general, an increase in uncertainty raises the probability that $Z_2 < Z^m$, implying that the set of outcomes $[Z^a, Z^m]$ where the wage constraint is binding has a larger probability of occurring. Expected profits fall when this effect more than offsets the positive impact on expected profits induced by the larger upside potential of productivity realizations over the interval $[Z^m, Z^b]$, which occurs only when the condition in (6) is verified.

To summarize, the constraint $\tilde{w}_2 = \max[w_2^{nash}, w_m]$ results in a concave profit function with respect to productivity. This generates a profit risk-premium, which is increasing in the variance of Z. By contrast, since with unconstrained Nash-bargaining the profit function is linear in Z, an increase in uncertainty does not affect the profit risk-premium, which is always nil.

We now discuss under what conditions higher uncertainty about Z_2 lowers the total stream of expected profits $(\overline{y}_1 - \overline{w}_1)$, the relevant flow value to understand the response of job creation.

Proposition 2 *When wage bargaining is constrained by the lower bound w_m, the total stream of expected profits falls when a mean-preserving spread of Z_2 lowers period 2 expected profits.*

Under Nash bargaining, $w_1^{nash} = w_2^{nash} = y(Z_t)/2 = Z_t$. Total profits are linear in productivity, and higher uncertainty about future productivity does not change the $t = 0$ value of a match to the firm, leaving the firm's vacancy posting decision unaffected. Full wage rigidity leads to the same conclusion.

The solution to constrained Nash bargaining implies $\tilde{w}_1^{nash} = w_1^{nash} + [E_1(J_2) - E_1(W_2)]/2$. Two observations are in order. First, since $\tilde{w}_1^{nash} \neq w_1^{nash}$, the wage payment in the first period differs from w_1^{nash} even when the wage constraint is not binding—intuitively, the non-zero probability of hitting the constraint in the second period affects the Nash surplus in the first period. Second, it is straightforward to verify that absent the constraint $w_1 \geqslant w_m$, both J_1 and W_1 would be identical to their respective values under unconstrained Nash bargaining. This result reflects the argument in Pissarides (2009), who shows that, as long as the wage of new hires is determined by the unconstrained Nash rule, wage determination in subsequent periods for a continuing match is irrelevant for job creation.

However, this result does not hold true when the constraint on wage bargaining is in place in *all* periods. That is, the requirement that $w_1 \geqslant w_m$ implies that at time 0 the firm anticipates the existence of states of the world in which it will not be possible to reduce w_1 sufficiently in response to a fall in $E_1(J_2)$. As a result, $\bar{y}_1 - \bar{w}_1$ falls, reducing job creation.

To conclude, notice that we derived the results of this section using a model where the vector of states does not include any endogenous variable. As a result, the response of expected profits to uncertainty shocks is independent of the state of the economy at $t-1$. By contrast, if the state of the economy affects the future path of endogenous variables, expected profits may fall by a larger amount when the $t-1$ level of productivity is low, compared to the case when level of productivity is high in $t-1$. In the general equilibrium model presented in the next section, a low value of Z_{t-1} increases the likelihood of a productivity draw $Z_t < Z^m$ for which the surplus share of the firm is lower. The Appendix discusses how the results for the simple model in this section can illustrate the state-dependence result we obtain in section 4.

4 General Equilibrium Model

4.1 Household Preferences

The economy is populated by a unit mass of atomistic, identical households. Each household is thought of as a large extended family containing a continuum of members along a unit interval. The household does not choose how many family members work; the measure of family members who work is determined by a labor matching process. Following Andolfatto (1996), Merz (1995), and much of the subsequent literature, we assume full consumption insurance between employed and unemployed individuals.

The representative household maximizes the expected intertemporal utility function

$$E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma} \right], \tag{7}$$

where C_t is aggregate consumption, $\beta \in (0,1)$ is the subjective discount factor, and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution.

4.2 Production

A unit mass of symmetric, perfectly competitive firms uses labor as the only input of production. To hire new workers, firms need to post vacancies, incurring a cost of κ units of consumption per vacancy posted. The probability of finding a worker depends on a constant-return-to-scale matching technology which converts aggregate unemployed workers, U_t, and aggregate vacancies, V_t, into the total number of new matches per period $M_t = \chi U_t^{\varepsilon} V_t^{1-\varepsilon}$, where $\chi > 0$ and $0 < \varepsilon < 1$. Each firm meets unemployed workers at a rate $q_t \equiv M_t/V_t$. As in Krause and Lubik (2007) and other studies, we assume that newly created matches become productive only in the next period. The inflow of new hires in $t+1$ is therefore $q_t V_t$.

Firms and workers separate exogenously with probability $\lambda \in (0,1)$.[11] Since separations can occur for existing productive matches or for matches which have not yet started production, the law of motion for employment L_t, summarizing the number of productive matches in the economy, is given by

$$L_t = (1-\lambda)\left(L_{t-1} + q_{t-1}V_{t-1}\right). \tag{8}$$

The number of unemployed workers searching for jobs is $U_t = 1 - L_t$. Each firm produces output according to the linear technology $Y_t = e^{Z_t} L_t$, where Z_t is exogenous aggregate productivity. We assume that Z_t follows a stationary autoregressive process:

$$Z_t = \rho_z Z_{t-1} + e^{\sigma_{Zt}} u_{zt},$$

where $u_{zt} \overset{i.i.d}{\sim} N(0,1)$ is an exogenous shock to the level of technology and σ_{Zt} captures exogenous second-moment or "uncertainty" shocks. Intuitively, when σ_{Zt} increases, there is higher uncertainty about the future time path of the stochastic process Z_t. We assume that σ_{Zt} follows the stationary autoregressive process

$$\sigma_{Zt} = \rho_\sigma \sigma_{Zt-1} + (1-\rho_\sigma)\sigma_Z + u_{\sigma t},$$

where $u_{\sigma t} \overset{i.i.d}{\sim} N\left(0, \sigma_\sigma^2\right)$ represents innovations to the volatility of the technology shock.[12]

[11] Hall (2005) and Shimer (2005) argue that the separation rate varies little over the business cycle, although part of the literature disputes this position; see Davis, Haltiwanger, and Schuh (1998) and Fujita and Ramey (2009). For search and matching models with endogenous separation see, among others, den Haan, Ramey, and Watson (2000), Merz (1995), and Walsh (2005).

[12] An alternative approach would be to assume that the variance of Z_t follows a GARCH process, such that σ_{Zt}^2 is a function of σ_{Zt-1}^2 and the squared scaled innovation to Z_t, u_{zt}^2. In this case, there would be only one shock, u_{zt}, driving the dynamics of the level and volatility of Z_t. For this reason, as pointed out by Fernández-Villaverde and Rubio-Ramírez (2010), GARCH models cannot be used to assess the effects of volatility shocks independently from the effects of level shocks.

Producers choose the number of vacancies and employment to maximize the expected present discounted value of their real profit stream subject to (8):

$$E_t \sum_{s=t}^{\infty} \beta_{t,t+s} \left(e^{Z_s} L_s - w_s L_s - \kappa V_s\right),$$

where $\beta_{t,t+1} \equiv \beta \left(C_{t+1}^{-\gamma}/C_t^{-\gamma}\right)$ denotes the stochastic discount factor of households, who are assumed to own the firms. Let J_t be the Lagrange multiplier associated to the constraint (8), corresponding to the value of a new match to the firm.

The first-order condition for vacancies equates the cost of filling a vacancy to the expected present discounted value of a filled vacancy:

$$\frac{\kappa}{q_t} = (1-\lambda) E_t \left(\beta_{t,t+1} J_{t+1}\right). \tag{9}$$

The first-order condition for L_t defines the value of hiring an additional worker to the firm:

$$J_t = e^{Z_t} - w_t + (1-\lambda) E_t \left(\beta_{t,t+1} J_{t+1}\right). \tag{10}$$

Intuitively, J_t is the per-period marginal value product of the match, net of the current wage bill, plus the expected present discounted continuation value of the match. By combining the two equations above, we obtain the following job creation equation:

$$\frac{\kappa}{q_t} = (1-\lambda) E_t \left\{\beta_{t,t+1} \left[e^{Z_{t+1}} - w_{t+1} + \frac{\kappa}{q_{t+1}}\right]\right\}. \tag{11}$$

Equation (11) states that, at the optimum, the vacancy creation cost incurred by the firm per current match is set equal to the expected discounted profit from the time-t match, $e^{Z_{t+1}} - w_{t+1}$, plus the expected discounted value of the vacancy creation cost per future match.

4.3 Wage Bargaining

In the benchmark search and matching model, the real wage splits the match surplus according to individual Nash bargaining. Let w_t^{nash} be the solution to the standard Nash-bargaining problem:

$$w_t^{nash} = \arg\max \left(J_t^{1-\eta} W_t^{\eta}\right), \tag{12}$$

where W_t represents the worker surplus, and $\eta \in (0,1)$ identifies the bargaining power of the worker. Using equations (9) and (10), the firm surplus J_t can be written as:

$$J_t = e^{Z_t} - w_t^{nash} + \frac{\kappa}{q_t}. \tag{13}$$

The worker's surplus W_t is the difference between the worker's asset value of being employed, H_t, and the value of being unemployed, $U_{u,t}$. The value of employment is given by the current wage plus the expected future value of being matched to the firm. With probability $1-\lambda$ the match will survive, while with probability λ the worker will become unemployed in $t+1$. As a result:

$$H_t = w_t^{nash} + E_t \left\{ \beta_{t,t+1} \left[(1-\lambda) H_{t+1} + \lambda U_{u,t+1} \right] \right\}. \tag{14}$$

The value of unemployment is given by:

$$U_{u,t} = b + E_t \left\{ \beta_{t,t+1} [p_t (1-\lambda) H_{t+1} + (1-(1-\lambda) p_t) U_{u,t+1}] \right\}, \tag{15}$$

where $p_t \equiv M_t/U_t$ denotes the probability of finding a job in period t and b is the real value of non-work activities performed when unemployed. Using (14) and (15), we can define the worker's surplus, $W_t \equiv H_t - U_{u,t}$:

$$W_t = w_t^{nash} - b + (1-\lambda)(1-p_t) E_t (\beta_{t,t+1} W_{t+1}). \tag{16}$$

The first-order condition with respect to w_t^{nash} in (12) implies the following sharing rule: $\eta J_t + (1-\eta) W_t = 0$. Using equations (13) and (16), it is possible to derive the wage payment under unconstrained Nash bargaining:

$$w_t^{nash} = \eta \left(e^{Z_t} + \kappa \frac{p_t}{q_t} \right) + (1-\eta) b. \tag{17}$$

We depart from the standard model by assuming that, in each period, the wage payment cannot fall below an exogenous level w_m. In this case, the solution to the Nash bargaining problem involves the occasionally binding constraint $w_t \geqslant w_m$. This implies that the equilibrium wage can no longer be written in terms of time-t variables only, since the time-s continuation values J_{t+s} and W_{t+s} take different values depending on whether the wage constraint is binding or not at time s. This also implies that even in periods when the outcome of Nash bargaining satisfies $w_t > w_m$, the Nash-bargained wage is different from w_t^{nash}.

The OBC on wage bargaining implies that the law of motion describing the rational equilibrium solution for the wage may be not be differentiable over some portions of the state space. As a result, standard perturbation methods cannot be applied to obtain the rational expectations solution of the model, since local approximations require the model equations to be differentiable over the state space, at least to an order commensurate with the degree of accuracy of the approximation.

To accommodate the use of perturbation methods, we solve the Nash bargaining problem subject to the OBC by adding to the objective function in equation (12) a term that prescribes a high cost for the violation of the constraint. This approach follows a well established methodology in the field of applied mathematics—the penalty function method—which converts the original optimization problem containing an inequality constraint into an unconstrained problem.[13] Under fairly general conditions, it is possible to prove that for a given objective function $f(\mathbf{x})$, a constrained set of the vector $\mathbf{x}$, and a given penalty function $\Gamma(\mathbf{x},\psi)$, the sequence of solutions to the optimization problem converges to the solution of the original problem when $\psi \to \infty$ (Luenberger, 1973). Moreover, since the penalty term in the unconstrained problem is a smooth function of the variables appearing in the original OBC, it is possible to apply standard perturbation techniques to approximate the model solution up to an arbitrary degree of accuracy.[14]

We modify the Nash bargaining problem in equation (12) by assuming that the Nash surplus is equal to $J_t^{1-\eta} W_t^{\eta} - \Gamma_t$, where $\Gamma_t \equiv \Gamma(w_t, \psi)$ is the continuous and differentiable penalty function. The term ψ parameterizes the speed at which the value of the penalty function increases as $w_t \to w_m$. The penalty function satisfies the following requirement:

$$\lim_{\psi \longrightarrow \infty} \Gamma(w_t, \psi) = \begin{cases} 0 & w_t \geqslant w_m \\ \infty & w_t < w_m \end{cases} . \tag{18}$$

While any wage in the bargaining set is ex ante feasible, the penalty function Γ_t implies that as $\psi \to \infty$ the firm and the worker never stipulate a wage payment that violates the constraint, since, in this case, the Nash surplus drops to minus infinity.

The first-order condition of the constrained Nash bargaining problem with respect to w_t implies

[13] Recent contributions that use a penalty function approach in dynamic stochastic general equilibrium models include, among others, Kim, Kollmann, and Kim (2010), Judd (1998), Preston and Roca (2007), and Rotemberg and Woodford (1999).

[14] Global solution methods provide an alternative approach to solve general equilibrium models featuring an OBC. However, the application of global methods is challenging in models with more than a limited number of state variables. This issue is particularly severe in models with stochastic volatility because the number of states doubles for each stochastic process that features time-varying volatility.

the following sharing rule:

$$\eta J_t + (\eta - 1) W_t - \Gamma_{w,t} J_t^{\eta} W_t^{1-\eta} = 0, \tag{19}$$

where $\Gamma_{w,t} \equiv \partial \Gamma_t / \partial w_t$. As shown in the Appendix, the above sharing rule results in the following wage schedule:

$$w_t = w_t^{nash} - \Lambda_t + (1 - \lambda)(1 - p_t) E_t (\beta_{t,t+1} \Lambda_{t+1}), \tag{20}$$

where w_t^{nash} is defined as in (17) and $\Lambda_t \equiv \Gamma_{w,t} J_t^{\eta} W_t^{1-\eta}$.

Equation (20) shows that the presence of the OBC affects the wage payment in two ways. First, productivity outcomes that imply a violation of the constraint result in large negative values of Λ_t. The corresponding surplus loss implies that in equilibrium the wage is above the unconstrained Nash wage. As $\psi \to \infty$, this wage approaches w_m. Second, even when the constraint is not binding at time t, the equilibrium wage is different from the wage implied by unconstrained Nash bargaining. The reason is that the worker's and firm's surplus depend on the present discounted value of the expected stream of future wage payments, which in turn is affected by the future values of the penalty function. As discussed in the previous section, as long as there is a positive probability that the wage constraint will bind in the future, the Nash surplus accounts for this change in the continuation value of the match relative to the unconstrained wage case. Other things equal, when the time-t expected present discounted value of Λ_{t+1} is more negative, i.e., when the firm and the worker expect that the economy will be operating closer to the wage constraint at time $t + 1$, w_t falls relative to its unconstrained Nash-bargained level. Intuitively, in such circumstances the firm and the worker stipulate a lower wage today to account for the fact that the wage constraint may be binding tomorrow.

4.4 Equilibrium

The per-period household's budget constraint is $C_t = w_t L_t + b(1 - L_t) + \Pi_t$, where Π_t are profits rebated to the household. In equilibrium, $\Pi_t = e^{Z_t} L_t - w_t L_t - \kappa V_t$, which implies the following aggregate resource constraint: $e^{Z_t} L_t + b(1 - L_t) = C_t + \kappa V_t$. The resource constraints states that total output plus home production must be equal to aggregate demand—the sum of consumption and the costs of posting vacancies. Table 1 presents the model equilibrium conditions, including the functional form for the derivative of the penalty function, $\Gamma_{w,t}$, discussed in Section 6.

5 Parameterization

We parameterize the model at quarterly frequencies and choose parameter values to match features of the U.S. economy reported in the business cycle literature. Variables without a time subscript denote steady-state values.

We set the discount factor, β, and the risk aversion coefficient, γ, at conventional values in the business cycle literature: $\beta = 0.99$ and $\gamma = 2$. For the stochastic volatility process, estimate a GARCH process on the TFP measure updated quarterly by the Federal Reserve Bank of San Francisco using the methodology in Basu and Fernald (XXXX). This implies a process for σ_{Z_t} with an estimated persistence $\rho_\sigma = 0.72$ and standard deviation $\sigma_\sigma = 0.32$. We choose the persistence and volatility of the stochastic productivity process to match the first-order autocorrelation and the absolute standard deviation of output observed in the data.[15] This requires setting $\rho_z = 0.9$ and $\sigma_Z = 0.0045$.

We set the elasticity of matches to unemployment, ε, equal to 0.4, in line with Blanchard and Diamond (1989). To maintain comparability with much of the existing literature, we assume the worker's bargaining power, η, is equal to ε. This choice satisfies the Hosios condition, implying that wages are set at the efficient level under unconstrained Nash bargaining. This parameterization is also consistent with the evidence in Flinn (2006), who estimates $\eta = 0.38$ for the U.S. Following Hagedorn and Manovskii (2008), we set the value of non-work activities, b, such that $b/w = 0.95$. This choice implies that under efficient Nash wage bargaining, i.e., in the absence of the OBC, the volatility of employment relative to output is approximately equal to one-half. The exogenous separation rate, λ, is 10 percent, in line with the evidence reported in Shimer (2005). Finally, the cost of vacancy posting, κ, and the matching efficiency parameter, χ, are chosen to match the quarterly, average job-finding probability and the quarterly, average probability of filling a vacancy, estimated respectively at 83 percent (Shimer, 2005) and 70 percent (den Haan, Ramey, and Watson, 2000). Table 2 summarizes the model parameters.

We choose our parameterization such that the model featuring unconstrained wage bargaining provides a reasonable benchmark to study the consequences of the OBC for the transmission of first- and second-moment shocks. This approach, in particular, motivates our choice for the flow value of unemployment, since it is well-known that lower values of the worker's outside option would result in a much smaller volatility of employment relative to output in the unconstrained economy. As a result, both first- and second-moment shocks would have by construction a negligible impact on job creation

[15] We consider the period 1972Q1 to 2010Q4, as in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012).

in the absence of the OBC. Moreover, even in the constrained economy, employment dynamics would be muted whenever the OBC has a low probability of becoming binding in the future.[16]

6 Solution Method

Since our primary focus is to study the effects of second-moments shocks, we approximate the model policy functions by computing a fourth-order Taylor expansion of the equilibrium conditions around the deterministic steady state.[17] Below, we refer to this approximation as the "unpruned" policy function. In addition, since locally approximated models can produce explosive simulations when approximated at second or higher order, we also compute "pruned" policy functions that discard the terms in the solution that have higher-order effects than the approximation order.[18] We use the pruning algorithm of Andreasen (2012) and Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2013). The Appendix provides full details about the pruned and unpruned policy functions.

In what follows, we discuss the computational approach we adopted to approximate the occasionally binding constraint with a penalty function. Additionally, we address some important differences between the pruned and unpruned state space that are relevant for the transmission of uncertainty shocks.

6.1 Penalty Function Approximation

A smooth penalty function – or, in our case, its first-derivative, $\Gamma_{w,t}$ – can be parameterized so that it approaches arbitrarily close the non-differentiable limiting function in (18), ensuring that in the limit the solution to the unconstrained maximization problem modified to include the penalty function converges to the solution of the original constrained problem. However, when the rational expectations equilibrium is obtained through a local Taylor expansion, the model policy functions corresponds to a polynomial approximation. Thus, regardless of the choice of the penalty function, the law of motion for $\Gamma_{w,t}$ is a polynomial of the same order as the order of the local approximation.

[16] We also provide results for the case of sticky wage adjustment. Obviously, in this model the wage constraint binds at all times.

[17] As pointed out by Andreasen (2012), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), Ruge-Murcia (2012), and van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012), at lower order of approximation, second-moment shocks do not enter independently in the approximated policy functions. At the first order, the policy functions are invariant to the volatility of the shock processes. At the second order, second-moment shocks only enter as cross-products with first-moment shocks.

[18] Kim, Kim, Schaumburg, and Sims (2008) first made this point in the context of a second-order approximation. They proposed pruning all terms of higher order, i.e., computing the quadratic term at $t+1$ by only squaring the first-order term from time t. This procedure has been extended to higher orders by various authors; see Andreasen (2012), Den Haan and De Wind (2012), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), Lan and Meyer-Gohde (2013), and Lombardo (2010).

The key issue is that even high-order Taylor approximations do not necessarily inherit properties such as monotonicity and convexity from the true underlying policy functions describing the dynamics of the model. In particular, even a fourth-order Taylor expansion of the penalty function may have poor properties, implying that the local approximation is inaccurate in regions of the state space that are of economic interest. Den Haan and De Wind (2012) and Brzoza-Brzezina, Kolasa, and Makarski (2013) show that this is the case for various benchmark models featuring non-negativity constraints approximated by penalty methods.

Instead of relying on local approximations of a given continuos and twice-differentiable penalty function, we propose an alternative approach that directly chooses a fourth-order polynomial which describes the first-derivative $\Gamma_{w,t}$ in the approximated law of motion of the model. To illustrate our approach, notice first that a Taylor expansion of order n guarantees that the approximation error is of order $n+1$ within the radius of convergence to the approximation point. However, alternative n-order polynomials can have a better performance in regions of the state space that are of economic interest, yet preserve a high order of accuracy in the neighborhood of the approximation point. We assume that $\Gamma_{w,t}$ is given by the fourth-order polynomial:

$$\Gamma_{w,t} = \sum_{i=0}^{4} \alpha_i \Gamma_i(w_t), \tag{21}$$

where $\Gamma_i(\cdot)$ is a polynomial basis function and α_i are parameters chosen to ensure that $\Gamma_{w,t}$ satisfies desired global properties over a given interval for w_t. We then compute the equilibrium law of motion of the model with a fourth-order Taylor expansion.

Several choices for the basis functions are possible, and alternative methods are available to select the coefficients α_i. Any approach requires specifying a grid of values for w_t and a corresponding set of values for the penalty $\Gamma_{w,t}$. We specify the criteria to select $\Gamma_{w,t}$ based on the distribution of wage outcomes in the model. Since the wage distribution is an endogenous equilibrium outcome, the procedure to estimate α_1, α_2, α_3, and α_4 is iterative, i.e., it requires solving the model with a given candidate function $\Gamma_{w,t}$ at every step, and verifying ex post whether $\Gamma_{w,t}$ meets the required criteria. We choose ordinary polynomials as basis functions and find the coefficients α_i using a least squares algorithm. The Appendix provides the details of the iterative procedure and it discusses alternative methods to select the coefficients α_i.

We parametrize $\Gamma_{w,t}$ to match the following properties. First, we require that $\Gamma_{w,t} \rightharpoondown 0$ for any wage such that $w_t \in \left[w, w^{95th}\right]$, where w is the steady-state wage and w^{95th} is the wage corresponding to 95^{th} percentile in the ergodic wage distribution in the model with unconstrained wage bargaining. This criterion ensures that $\Gamma_{w,t}$ is approximately equal to zero for a large set of wage outcomes when the OBC is not binding. Second, we require that the OBC eliminates a given fraction of the wage outcomes observed in the unconstrained model ergodic wage distribution. Since in the data such target is not directly observable, we use the estimates in Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward, 2007 who document that in a panel of OECD countries an average of 26 percent of wage adjustments are subject to downward real wage rigidity, in the sense that 26 percent of real wage cuts that would have taken place in an unconstrained economy are prevented by the rigidity in wage contracting. Accordingly, we assume that the OBC eliminates 26 percent of the wage outcomes observed in the left tail of the ergodic wage distribution in the unconstrained economy.[19] This target endogenously implies that the OBC is binding on average 15 percent of the time. As shown below, while the OBC appears to eliminate a relatively small share of the possible wage outcomes, it affects wage-setting and employment dynamics for a sizable portion of the state space through its impact on the expected future stream of profits from a match when $w_t > w_m$.

In the Appendix, we study how alternative targets for the fraction of wage outcomes eliminated by the OBC affect our results. We also assess the accuracy of the approximated solution by computing Euler equation errors. Moreover, we compare the performance of the penalty function in (21) relative to the case in which $\Gamma_{w,t}$ is obtained using a fourth-order approximation of an exponential function parametrized to return the same target values for $\Gamma_{w,t}$ we use to find the optimized coefficients α_i. We inspect the behavior of the two approximated functions with respect to their argument w_t —a partial-equilibrium comparison—as well as the general equilibrium dynamics of w_t implied by each method.

6.2 Pruned and Non-Pruned Approximations

Lombardo and Uhlig (2014) show that pruned polynomial expansions can be interpreted as Taylor expansions for an appropriately chosen domain and an approximation point of the original function. Therefore, since both pruned and unpruned solutions rely on the Taylor theorem, the relative level of accuracy depends on the specific model at hand as well as on the kind of nonlinearity that the approximation attempts to capture. We discuss in the following why a fourth-order aproximation is necessary to discuss the impact of uncertainty shocks in a model with state-dependence.

[19] When computing the two ergodic wage distributions—with and without the OBC—we parametrize the productivity process such that each model reproduces the observed volatility of output.

Nevertheless, pruning the state space has important consequences for the transmission of second-moment shocks. As shown in Cacciatore and Ravenna (2015), the third-order, pruned state space does not preserve some fundamental properties that characterize the non-linear equilibrium conditions of the model. In particular, the effects of second-moments shocks becomes *history-independent* in the pruned state space, since past shocks play no role for the propagation of second-moment shocks. This result may seem surprising, since the use of higher-order perturbation techniques implies that the model solution is nonlinear. To gain intuition, notice that, up to the third order, state dependence reflects two sources of nonlinearity in the unpruned state space. First, the policy functions feature terms that interact current shocks with previous states. Second, there are terms that are nonlinear in the states alone. The evolution of the states alone induces history dependence of second-moment shocks because the third-order increment of the states (which responds to both first- and second-moment shocks) multiplies the first- and second-order increment of the states (which only respond to first-moment shocks).[20]

As discussed by Cacciatore and Ravenna (2015), up to the third order, the coefficients of the nonlinear terms interacting the state variables with volatility shocks are always zero, unless the term also multiplies first-moment shocks, which are zero on average. For this reason, the *impact* effect of volatility shocks cannot be state-dependent, both in the pruned and unpruned state space. History dependence can then arise only because of the second channel discussed above, the endogenous evolutions of the states. However, in the pruned state space, the third-order increment of the state variables (the only increment affected by second moment shocks) never interacts with the first- and second-order increments of the state variables, eliminating state dependence. As a consequence, pruning can capture state-dependent dynamics in response to second-moment shocks only in fourth- or higher-order approximations. The above discussion can be summarized by the following two propositions:

Proposition 3 *Up to the third order, the history of past shocks does not affect the impact response of endogenous variables to second-moments shocks in the unpruned state space.*

Proposition 4 *Up to the third order, the history of past shocks does not affect the response of endogenous variables to second-moment shocks at any horizon in the unpruned state space*

In what follows we only use the unpruned state space when analyzing the consequences of time-varying volatility.

[20] As shown in the Appendix, the third-order policy functions can be written as the sum of the first-order approximation (the first-order increment) and the second- and third-order increments (the difference between two approximations of adjacent order).

7 Results

7.1 Wage Dynamics and Ergodic Wage distribution

To illustrate the consequences of the OBC on wage bargaining we conduct two experiments. First, we compare the impact response of wages following positive and negative productivity shocks of different size. Second, we compute the ergodic wage distribution. In both cases, we compare wage dynamics in the economy subject to the OBC to those implied by unconstrained Nash bargaining.

Figure 1 plots the wage impact response to productivity shocks from a range $-2\sigma_Z$ to $2\sigma_Z$. We generate impulse responses at the ergodic mean in the absence of shocks, labeled by Juillard and Kamenik (2005) the "stochastic steady state."[21] The impact response is defined as the percentage deviation of the wage from the stochastic steady state. In the economy with the OBC (left panel), the wage never falls below 1 percent, the floor implied by our parametrization of the penalty function. By contrast, for positive productivity innovations, the wage response is similar to the response of the unconstrained wage (right panel).

Figures 2 and 3 present the ergodic wage distribution for the economy with the OBC and the unconstrained economy. In both cases, the ergodic distribution is obtained by pooling 1000 simulations with length equal to 250 periods. To assure non-explosive behavior of the simulations, we use the pruned approximation to the policy functions in this experiment. The penalty function skews the wage distribution towards the wage constraint, and prevents the wage from following below w_m. The skewness of the constrained-wage distribution is equal to 0.9, and about 4 times as large relative to the unconstrained-wage distribution, which ha a skewness equal to 0.23.

7.2 Business Cycle Dynamics with Occasionally Binding Constraints on Wage Bargaining

This section discusses how the presence of the OBC affects the propagation of productivity shocks, Z_t. We consider three alternatives: The model with the OBC, the model with unconstrained Nash bargaining, and a model with symmetric real wage rigidity as in Hall (2005). The three models differ only with respect to the wage equation that splits the surplus between the worker and the firm. In

[21] The ergodic mean in the absence of shocks is the fixed point of the third-order approximated policy functions. It is obtained by simulating the system with all shocks set to 0 for all time periods iterating forward until convergence. Our results are not significantly affected if we assume that the economy is at the ergodic steady state, defined as the mean of the ergodic distribution implied by the law of motion for the vector of state and control variables. We verified that assessing the impact of the shocks starting at the ergodic mean and building generalized impulse responses (accounting for the nonlinearity of the dynamics in the size of the shock, the point in the state space where the shock occurs, and the distribution of future random shocks) does not affect our results. See the Appendix for details.

the model with the OBC, the wage is determined as in equation (20). Unconstrained Nash bargaining implies that $w_t = w_t^{nash}$, as in equation (17) . Hall's wage rigidity implies that w_t is a weighted average of the unconstrained Nash wage, w_t^{nash}, and the steady-state wage, w, i.e., $w_t = (1-\xi)\, w_t^{nash} + \xi w$, where we set $\xi = 0.5$.

Figure 4 plots the response of key macroeconomic variables following a reduction in Z_t equal to one standard deviation, and assuming that the economy is at the stochastic steady state.[22] Lower productivity reduces the present discounted value of new and existing matches, which, other things equal, reduces vacancy posting. As pointed out by Shimer (2005) and Hall (2005), in search models of the labor market the change in employment L_t following a productivity shock is larger the smaller the elasticity of wages to productivity. As shown in Figure 4, this elasticity is small in the presence of wage rigidity (dotted line) and relatively large with unconstrained Nash bargaining (dashed line). Consider now the economy with the OBC (continuous line). Employment L_t falls approximately by 2 percent relative to the initial steady state, while the reduction is approximately equal to 0.25 percent in the unconstrained-wage scenario. The negative productivity shock increases the likelihood that the wage constraint will bind in the future. This in turn leads to a larger reduction in the expected firm's surplus since the time-t wage cannot be lowered sufficiently to compensate for the risk that the wage-negotiation at future time t may be constrained by the OBC . As a result, vacancy posting responds more strongly, with negative consequences for output and employment. At the same time, the OBC implies a smaller drop in employment relative to the scenario with symmetric wage rigidity. In the latter case, the constraint on wage adjustment is binding in all periods, further reducing the expected profit from a match. However, as we discuss next, this result does not imply that rigid wages also result in a larger impact of uncertainty shocks.

7.3 The Impact of Uncertainty Shocks

We now turn to the consequences of a persistent increase in uncertainty. We assume that the standard deviation of the productivity innovation, σ_{Zt}, increases by two standard deviations, corresponding to an 80 percent increase relative to the steady-state. We generate impulse responses to σ_{Z_t} in order to measure the *pure uncertainty effect* resulting from a change in the distribution of future shocks, for given Z_t. To this end, we net out *realized uncertainty effect*, that is, the impact of σ_{Zt} on the current realization of productivity, as in Basu and Bundick (2012), Born and Pfeifer (2014), and Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011).

[22] Our results are unaffected if we consider generalized impulse responses.

Figure 5 presents the impulse responses for the model with unconstrained Nash bargaining (dashed line) and Hall's wage rigid (dotted line), assuming again that the economy is at the stochastic steady state. The uncertainty shock is recessionary in both cases: Employment, output, and consumption fall when σ_{Zt} increases. However, the quantitative impact of higher uncertainty is very modest. At the trough, L_t falls by 0.01 percentage points when the wage payment is determined by unconstrained, per-period wage bargaining. With Hall's wage rigidity, L_t falls by 0.03 percentage points.

To understand these results, recall that higher uncertainty raises the probability of more extreme realizations of future productivity innovation, both upward and downward. The response of firms' and workers' expectations to higher risk determines how increased uncertainty affects macroeconomic dynamics. As illustrated in Section 2, the impact of a change in uncertainty depends on the degree of concavity of the discounted stream of future profits over the state space. Greater concavity results in a larger and more variable profit-risk premium, leading to a stronger response of expected profits and job creation. For a plausible parameterization of the model, unconstrained wage bargaining and wage rigidity imply that the response of the profit-risk premium to higher uncertainty is muted. Put differently, an increase in the likelihood of more extreme first-moment shocks does not bring about a large change in the expected firm's profits.

This lack of sensitivity of job creation to changes in uncertainty is captured by the elasticity of new matches M_t to the standard deviation of productivity, $\omega_{M,\sigma}$, evaluated at the stochastic steady state. The fourth-order policy function for M_t implies that $\omega_{M,\sigma} = -0.00029$ in the model with unconstrained wage bargaining and $\omega_{M,\sigma} = -0.00088$ in the model with wage rigidity.[23] In relative terms, wage rigidity amplifies the response of employment relative to the unconstrained scenario. However, the effect is quantitatively very small.

Figure 6 presents the impulse responses to the same uncertainty shock for the model with the OBC on wage bargaining. When uncertainty increases at the stochastic steady state, the response of output and employment (continuous lines) is more than five times larger compared to the model with rigid wages. At the trough, L_t falls by approximately 0.25 percentage points. Thus, while the employment response to first-moment shocks is smaller in the model with the OBC relative to the rigid wage scenario, the opposite is true for second-moment shocks. The OBC results in a sizable profit risk-premium which is increasing in the variance of future shocks, since the expected fall in profits associated to deeper recessions is larger than the expected gain in profits associated to stronger

[23] Since we assume that uncertainty increases when the economy is at the ergodic mean in the absence of shocks, the impact response of employment relative to the stochastic steady state is equal to the product between the elasticity $\omega_{M,\sigma}$ and the size of the uncertainty shock, $u_{\sigma t}$.

expansions.

The presence of the OBC has further implications for the propagation of uncertainty shocks, since it implies that a given increase in uncertainty has different consequences depending on the prevailing state of the business cycle. To investigate the importance of state-dependence, we perform the following experiment. Consider the same uncertainty shock described above, but assume that the economy is either in an expansion or in a recession when uncertainty increases. To fix ideas, suppose that, starting from the stochastic steady state, the level of productivity increases (decreases) by 3 standard deviations at time $t-1$. Assume also that uncertainty increases at time t.[24] Figure 6 plots again the "pure effect" of the uncertainty shock, since we subtract to the dynamics of each variable the response to the same productivity innovation u_{zt-1} keeping uncertainty constant. The dashed (dotted) line in Figure 6 plots the impulse responses for the case in which uncertainty increases during an economic expansion (recession). At the trough, the uncertainty shock lowers employment by approximately 0.55 percentage points when the economy is in a recession. This figure is approximately 10 times larger relative to what is observed in the economic expansion. Intuitively, the change in the profit risk-premium is smaller in the expansion, since the likelihood that the wage constraint will bind in the future is smaller.[25] As a result, higher dispersion of future productivity shocks has a negligible impact on the expected profit from a match. Moreover, the size of the upturn is virtually irrelevant for the propagation of uncertainty shocks during an economic expansion. By contrast, in the recession the economy already operates close to the wage constraint. Therefore, higher uncertainty induces a more sizable drop in the firm's expected profit, leading to a stronger fall in job creation.

Finally, the OBC implies that the wage response need not be proportional to the size of the employment change. The current wage only reflects the general equilibrium response of Nash bargaining given the current total surplus, which depends on the distribution of future shocks. Thus, similar wage dynamics may be consistent with different employment outcomes.

7.4 Understanding the Dynamics of Measured Uncertainty

The empirical literature has proposed various indicators of macroeconomic uncertainty. A popular measure is the so-called subjective forecast uncertainty, capturing the uncertainty of forecasters about

[24] By assuming that the innovations to Z_t and to σ_t are realized in two consecutive periods, we implicitly set to zero the higher-order terms interacting the innovation in uncertainty, $u_{\sigma t}$, and the innovation to the first-moment of the productivity process, u_{zt-1}. Thus, we continue to isolate the impact of higher expected variance of shocks, without letting increased volatility affect the ex-post realizations of technology shocks.

[25] Recall that the discount rate for the future value of a match depends on both the stochastic discount factor, and the probability $(1-\lambda)$ of the match continuing in the next period.

their own forecasts. This measure corresponds to the conditional variance of future shocks that are unforecastable from the perspective of the agents. For instance, as discussed by Bloom (2014), since 1992 the Federal Reserve Bank of Philadelphia has been asking forecasters to provide probabilities for GDP growth falling into ten different bins around their median forecast. Forecasters' uncertainty, measured by the median within forecasters' subjective conditional volatility, more than doubled when economic growth fell sharply in 2008-2009. In the 2001 recession, it rose by 50 percent. Jurado, Ludvigson, and Ng (2013) construct an alternative measure of uncertainty that exploits the same notion of uncertainty as conditional variance of the forecast error. They measure forecast errors in a system of forecasting equations that uses data on hundreds of monthly economic series. Jurado, Ludvigson, and Ng (2013) show that this measure of uncertainty rises dramatically in large U.S. recessions. Orlik and Veldkamp (2014) report stronger time-variation of the forecast error variance when uncertainty is measured using survey data rather than GDP volatility.

The existence of an OBC can account for the correlation between recessions and the conditional variance of the forecast error. In particular, the conditional variance of the output forecast error can be time-varying even if the conditional volatility of shocks that is unforecastable from the perspective of economic agents has not changed. To illustrate this point, consider the following experiment. Assume that the conditional volatility of productivity shocks is constant, i.e., assume that there are no uncertainty shocks. Starting from the ergodic mean in the absence of shocks, suppose that at time t a productivity innovation occurs.[26] At this point, compute the output forecast error variance conditional on the information set I_t:

$$\sigma^*_{t+1} \equiv E\left[\left(Y_{t+1} - E\left(Y_{t+1}|I_t\right)\right)^2|I_t\right].$$

To compute σ^*_{t+1} we simulate the model drawing 5000 innovations, obtaining a distribution for Y_{t+1}. We repeat the same exercise for different initial productivity shocks at time t, ranging from $-2\sigma_Z$ to $2\sigma_Z$. As shown by Figure 8, the distribution of Y_{t+1} is more dispersed when the economy is in a recession. Intuitively, agents anticipate that job creation responds more strongly to future innovations in states of the world in which the wage constraint is binding, and this region is larger when the economy is operating close to the constraint. By contrast, in good states of the world the economy operates far away from the wage constraint and expected outcomes are less dispersed. As a result, the forecast error variance σ^*_{t+1} is state dependent, increasing in recessions and shrinking during

[26] In the Appendix, we show that our results are unaffected if we assume that the economy starts at the ergodic mean.

expansions. For instance, $\sigma^*_{t+1} = 0.0106$ when Y_t is 1 percentage point above its ergodic mean, while $\sigma^*_{t+1} = 0.0196$ when Y_t is 1 percentage point below its ergodic mean.

Figure 8 shows that during a recession the distribution of Y_{t+1} is more dispersed both in the right and left tail relative to what is observed during an expansion. Moreover, in the expansion, the distribution of Y_{t+1} still displays right-skewness. To understand these results, it is useful to inspect the distribution of Y_{t+1} implied by the model with unconstrained wage bargaining. As shown in Figure 9, the distribution of Y_{t+1} is approximately identical in the recession and in the economic expansion. Additionally, in neither case the distribution of Y_{t+1} presents the same right-skewness as in the case of the model with the OBC. This suggests that the higher dispersion of output observed in both tails of the distribution in a recession is the results of the OBC rather than of other nonlinearities in the model. Under constrained Nash bargaining, a positive productivity shock decreases the probability that the wage constraint will bind in the future by more in a recession compared to an expansion. As a consequence, in a recession the distribution of future output realizations has relatively more outcomes both in the right and left tail compared to an expansion. Finally, the two figures suggest that the right-skewness of future output realizations observed in the expansion is also the consequence of the OBC. Intuitively, negative productivity shocks increase the probability that the wage constraint will bind in the future even in good times, inducing an asymmetric distribution in the $t + 1$ outcomes for both employment and output.

8 Conclusions

We have shown that wage adjustment in frictional labor markets plays a central role in propagating exogenous uncertainty shocks and in explaining the observed countercyclicality of empirical measures of aggregate uncertainty. The existence of an occasionally binding constraints in wage negotiations introduces a key non-linearity in the firm's profit function, since the reduction in the present discounted value of job creation is significantly larger in recessions compared to its increase in expansions. In turn, such non-linearity results in a large profit-risk premium when uncertainty increases at times of low aggregated demand, inducing large and state-dependent effects of exogenous uncertainty shocks. Importantly, the propagation of uncertainty shocks through the wage channel is not a function of the size of the inefficiency in wage setting, but of the degree of concavity in the resulting profit function.

The occasionally binding constraint in wage negotiations also implies that the variance of the unforecastable component of future economic outcomes always increases at times of low economic activity. Thus, measured uncertainty increases during recessions even in the absence of exogenous uncertainty shocks. This result has important implications for the identification of uncertainty shocks in reduced-form econometric models.

A methodological contribution of this paper involves the solution of general equilibrium models that feature stochastic volatility and occasionally binding constraints by mean of local approximations.

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TABLE 1: MODEL EQUATIONS

(1)	$L_t = (1-\lambda)(L_{t-1} + M_{t-1})$
(2)	$M_t = \chi U_t^{\varepsilon} V_t^{1-\varepsilon}$
(3)	$U_t = 1 - L_t$
(4)	$\frac{\kappa}{q_t} = (1-\lambda)\beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} J_{t+1} \right]$
(5)	$\eta J_t + (\eta - 1) W_t - \Gamma_{w,t} J_t^{\eta} W_t^{1-\eta} = 0$
(6)	$q_t = M_t / V_t$
(7)	$p_t = M_t / U_t$
(8)	$J_t = e^{Z_t} - w_t + \frac{\kappa}{q_t}$
(9)	$W_t = w_t - b + (1-\lambda)(1-i_t)\beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} W_{t+1} \right]$
(10)	$\Gamma_{w,t} = \alpha_1 (w_t - w_m) + \alpha_2 (w_t - w_m)^2 + \alpha_3 (w_t - w_m)^3 + \omega$
(11)	$e^{Z_t} L_t + b(1 - L_t) = C_t + \kappa V_t$
(12)	$Z_t = \rho_z Z_{t-1} + e^{\sigma_{Z t}} u_{zt}$
(13)	$\sigma_{Zt} = \rho_\sigma \sigma_{Zt-1} + (1-\rho_\sigma)\sigma_Z + u_{\sigma t}$

TABLE 2: MODEL PARAMETERS

Risk aversion	$\gamma = 2$
Discount factor	$\beta = 0.99$
Matching function elasticity	$\varepsilon = 0.4$
Workers' bargaining power	$\eta = 0.4$
Home production	$b = 0.94$
Matching efficiency	$\chi = 0.87$
Vacancy cost	$k = 0.072$
Exogenous separation rate	$\lambda = 0.1$
TFP Level innovation, std. deviation	$\sigma_z = 0.0035$
TFP Level, persistence	$\rho_z = 0.95$
TFP Volatility innovation, std. deviation	$\sigma_\sigma = 0.76$
TFP Volatility, persistence	$\rho_\sigma = 0.88$

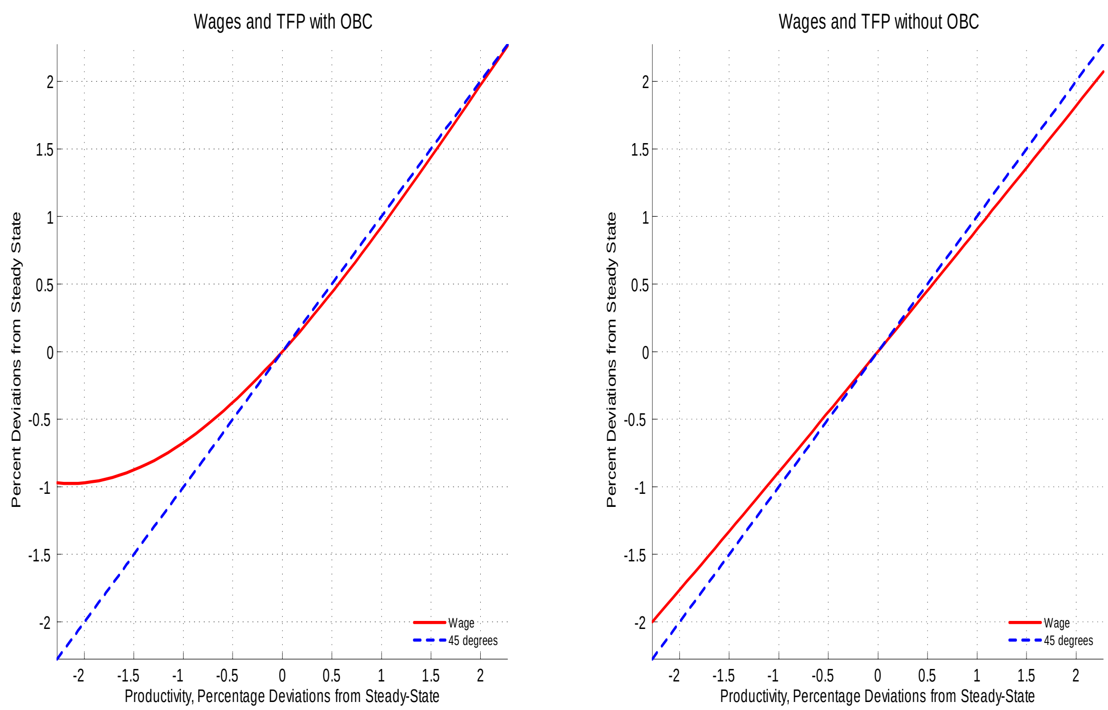

Figure 1. Impact response of the real wage (vertical axes) following productivity shocks of different size (horizontal axis). *Continuous line*: occasionally binding constraint on the real wage; *Dashed line*: unconstrained Nash wage bargaining; *Dotted line*: 45 degrees line. The economy is at the stochastic steady prior to the realization of productivity shocks. Solution method: unpruned, third-order approximation of the policy functions.

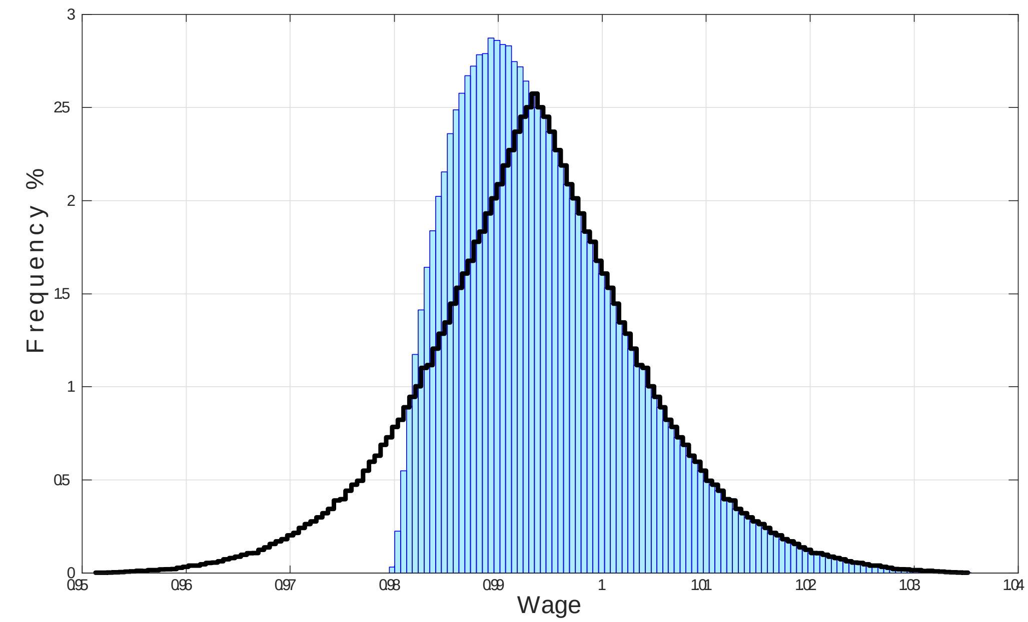

Figure 2. Wage distribution, occasionally binding constraint on the real wage. The distribution is obtained by simulation of the unpruned, third-order policy functions using only TFP shocks. Stairs plot shows the distribution that would obtain if wage realizations above the 50th percentile were symmetric around the median.

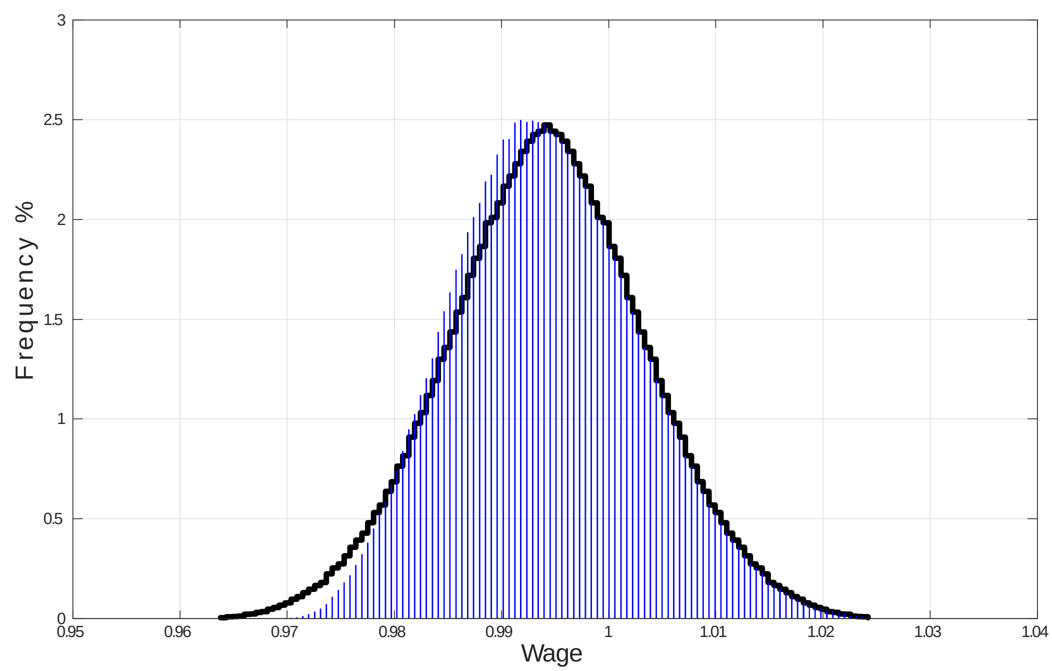

Figure 3. Wage distribution, unconstrained Nash bargaining. The distribution is obtained by simulation of the unpruned, third-order policy functions using only TFP shocks. Stairs plot shows the distribution that would obtain if wage realizations above the 50th percentile were symmetric around the median.

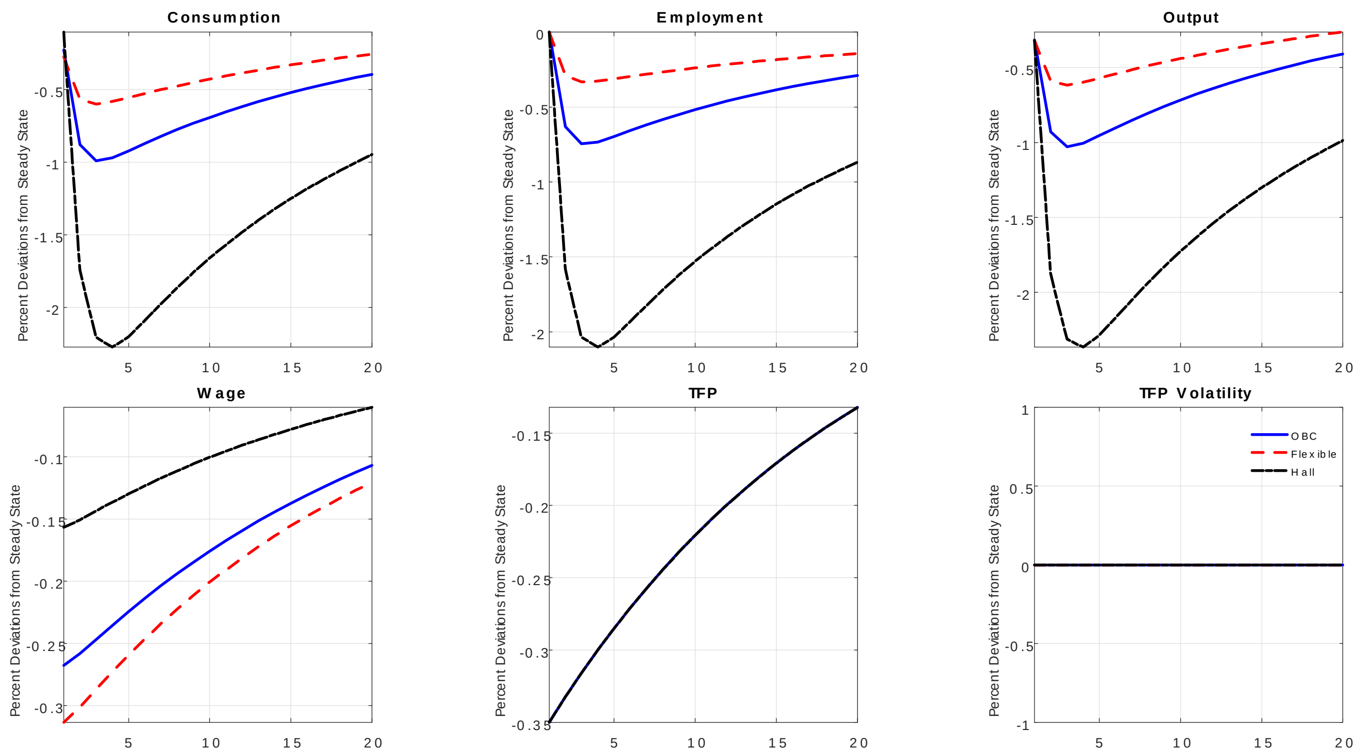

Figure 4. Impulse responses, one standard deviation increase in productivity. *Solid line*: occasionally binding constraint on the real wage; *Dashed line*: fully flexible per-period Nash bargaining; *Dotted line*: Hall (2005) real wage rigidity. The economy is at the stochastic steady prior to the realization of the productivity shock. Model solution: unpruned, third-order approximation of the policy functions.

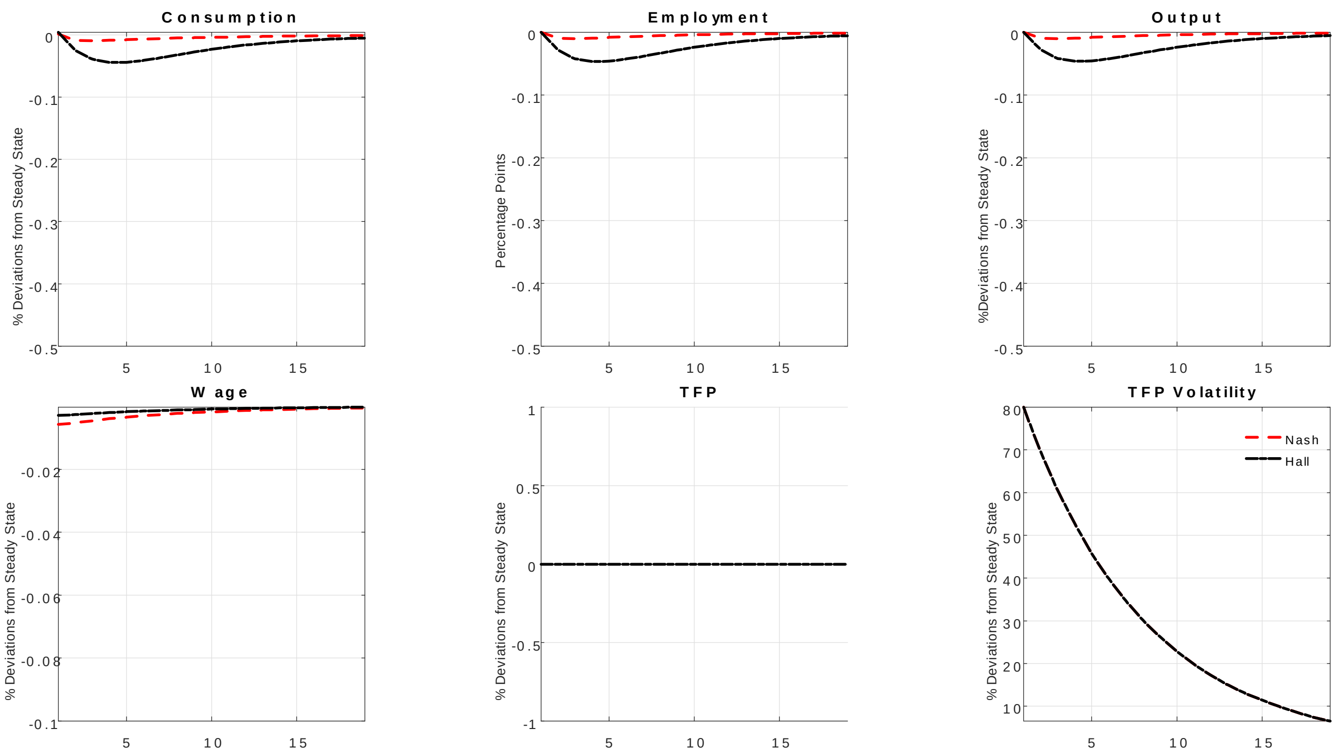

Figure 5. Impulse responses, one standard deviation increase in uncertainty. *Solid line:* flexible Nash wage bargaining; *Dotted line*: Hall (2005) wage rigidity. The economy is at the stochastic steady prior to the realization of the productivity shock. Solution method: unpruned, third-order approximation of the policy functions.

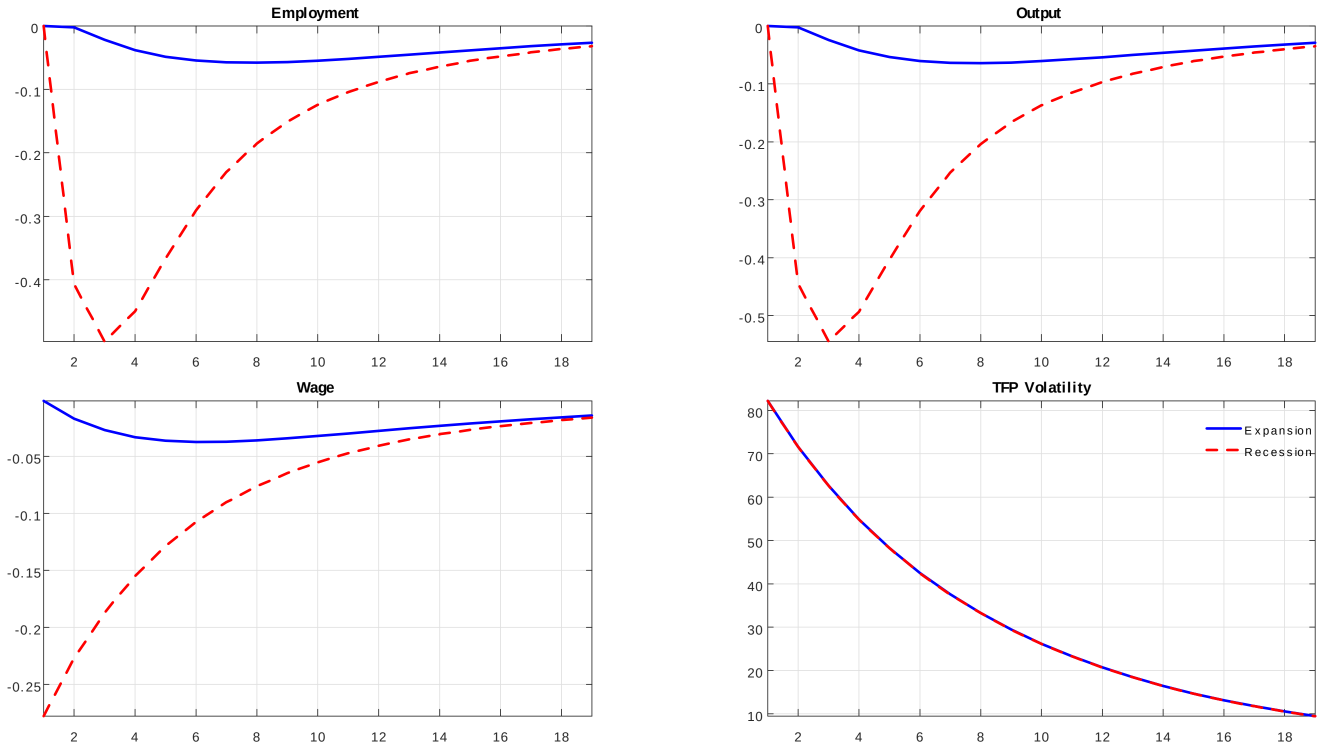

Figure 6. Net impact of uncertainty shocks over the business cycle. *Solid line*: expansion; *Dotted line:* recession. Expansion (recession): one standard deviation increase (reduction) in the level of productivity. We assume a one standard deviation increase in uncertainty in the quarter that follows the productivity shock. For any given variable y, we plot the difference between the percentage change in y (relative to the stochastic steady state) when both productivity and uncertainty shocks are realized and the percentage change in y absent the uncertainty shock. The economy is at the stochastic steady prior to the realization of the productivity shock. Solution method: unpruned, third-order policy functions.

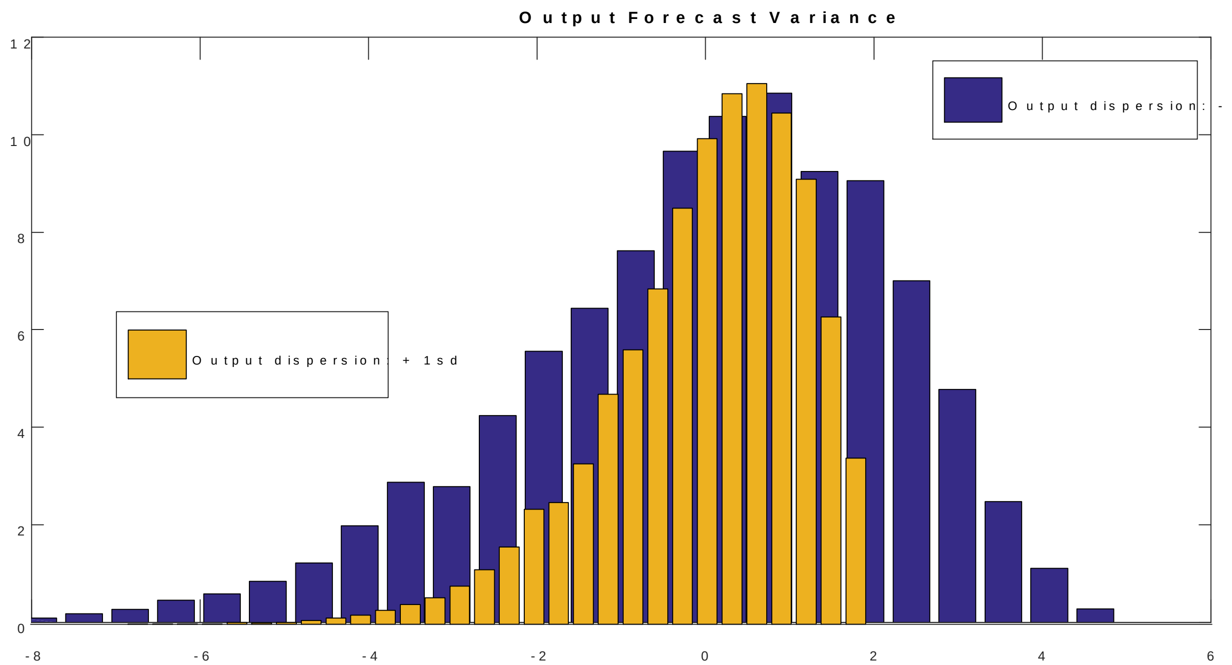

Figure 8. Distribution of output realizations at time $t+1$following a negative (dark bars) and positive (light bars) one-standard deviation TFP shock at time t. Outcomes are in percentage deviations from the level of output at time t.

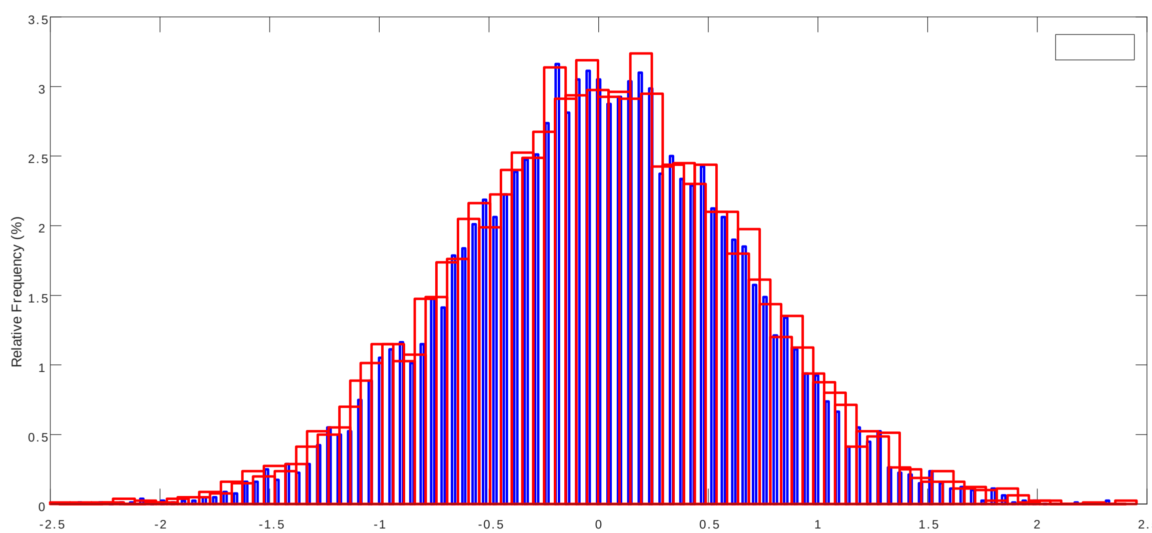

Figure 9. Distribution of output realizations at time $t+1$following a negative (dark bars) and positive (light bars) one-standard deviation TFP shock at time t. Model with unconstrained Nash bargaining. Outcomes are in percentage deviations from the level of output at time t.

Appendix

A Partial-Equilibrium Search and Matching Model

A.1 Proof for Proposition 1

Part 1: Under flexible wages, an increase in the variance of Z_2 holding constant $E(Z_2)$ does not affect period-2 expected profits

In the case of completely flexible wages, our assumptions on the distribution of Z_2, on $y(Z_2)$, and on Nash bargaining imply:

$$\begin{aligned} E_1(J_2) &= \int\limits_{Z\in[Z^a,Z^b]} sdF(Z) \\ &= E_1(Z_2) \end{aligned}$$

Since the increase in uncertainty for Z_2 holds $E_1(Z_2)$ constant, $E_1(J_2)$ is also constant.

Part 2: Under fully rigid wages, an increase in the variance of Z_2 holding constant $E(Z_2)$ does not affect period-2 expected profits

When wages are completely rigid and do not respond to productivity, the constant-wage constraint is binding over all the support of Z.This implies that a constant wage set at any level delivers an expected period-2 profit that depends exclusively on the expected level of productivity: $E_1(J_2) = \int_{Z^a}^{Z^b} [y(Z) - w_{const}]\, dF(Z)$. Since the increase in uncertainty for Z_2 holds $E_1(Z_2)$ constant and thus $E_1[y(Z)]$ constant, $E_1(J_2)$ is also constant. The elasticity of profits with respect to Z is constant, regardless of the wage choice. The realized and expected level of profit are instead a function of the wage choice w_{const}:

$$E_1(J_2) = 2E(Z) - w_{const}.$$

Part 3: An increase in the variance of Z_2 holding constant $E(Z_2)$ lowers period-2 expected profits if wage bargaining is constrained by a lower bound w_m for wages.

Under constrained-Nash bargaining, the expected period-2 profit can be written as:

$$E_1(J_2) = \int\limits_{Z^a}^{Z^m} [y_2(Z_2) - w_m]\, dF(Z) + \int\limits_{Z^m}^{Z^b} \frac{y_2(Z_2)}{2} dF(Z) \tag{A-22}$$

$$= \int\limits_{Z^a}^{Z^m} [y_2(Z_2) - w_m]\, dF(Z) + \int\limits_{Z^m}^{Z^b} sdF(Z) \tag{A-23}$$

$$= \int\limits_{Z^a}^{Z^m} 2sdF(Z) - \int\limits_{Z^a}^{Z^m} w_m dF(Z) + \int\limits_{Z^m}^{Z^b} sdF(Z) \tag{A-24}$$

Define

$$\begin{aligned} \sigma &= Z^b - Z^a \\ \overline{\sigma} &= \overline{Z}^b - \overline{Z}^a \end{aligned}$$

where $\overline{Z}^a, \overline{Z}^b$ are two reference points on the support of Z such that $Z^a < \overline{Z}^a < \overline{Z}^b < Z^b$ and $Z^a - \overline{Z}^a = \overline{Z}^b - Z^b$. This implies that $\Delta\sigma = \sigma - \overline{\sigma}$ defines an increase in variance which keeps the expected value of Z constant. Then we can rewrite the extremes of the support as a function of the dispersion measure:

$$Z^a = \overline{Z}^a - \frac{1}{2}(\sigma - \overline{\sigma})$$
$$Z^b = \overline{Z}^b + \frac{1}{2}(\sigma - \overline{\sigma})$$

Note that for a uniform distribution, the range σ is related to the variance of the distribution by the relationship $Var(Z) = 1/12\sigma^2$.The introduction of the variable $\Delta\sigma$ lets us evaluate the derivative of expected profits with respect to a simultaneous change in both Z^a and Z^b as a function of the range σ.

Consider the second integral in equation (A-23). Evaluating the integral we obtain:

$$\begin{aligned}
&\int_{Z^m}^{Z^b} Z \frac{1}{Z^b - Z^a} dZ \\
&= \frac{1}{\sigma}\frac{1}{2}\left[\left(\overline{Z}^b + \frac{1}{2}\Delta\sigma\right)^2 - Z^{m^2}\right] \\
&= \frac{1}{2}\overline{Z}^{b^2}\sigma^{-1} + \frac{1}{8}\sigma + \frac{1}{8}\overline{\sigma}^2\sigma^{-1} - \frac{1}{4}\overline{\sigma} + \frac{1}{2}\overline{Z}^b - \frac{1}{2}\overline{Z}^b\overline{\sigma}\sigma^{-1} - \frac{1}{2}Z^{m^2}\sigma^{-1}
\end{aligned}$$

To assess the marginal impact of an increase in the dispersion σ, evaluate the derivative of the latter expression:

$$\begin{aligned}
\partial/\partial\sigma &= -\frac{1}{2}\overline{Z}^{b^2}\sigma^{-2} + \frac{1}{8} - \frac{1}{8}\overline{\sigma}^2\sigma^{-2} + \frac{1}{2}\overline{Z}^b\overline{\sigma}\sigma^{-2} + \frac{1}{2}Z^{m^2}\sigma^{-2} \\
&= -\frac{1}{2}\frac{1}{\sigma^2}\left[\frac{1}{4}\left(\overline{Z}^b + \overline{Z}^a\right)^2 - Z^{m^2}\right] + \frac{1}{8} \\
&= -\frac{1}{2}\frac{1}{\sigma^2}\left[E\left\{Z\right\}^2 - Z^{m^2}\right] + \frac{1}{8} \qquad \text{(A-25)}
\end{aligned}$$

Equation (A-25) implies that $\partial/\partial\sigma < 0$ if $\left[E\left\{Z\right\}^2 - Z^{m^2}\right] > \frac{\sigma^2}{4}$. The equation (A-25) has a useful interpretation. Assume that the value of the first integral in equation (A-23) is always equal to zero. This would imply that when the Nash bargained wage violates the constraint $w_t \geqslant w_m$ (or, equivalently, $Z < Z^m$) all the surplus is assigned to the worker. Then equation (A-25) is equal to $\partial E_1(J_2)/\partial\sigma$. In this case, a sufficient condition for $E_1(J_2)$ to fall when the dispersion of Z increases is that Z^m is sufficiently below the average productivity level (and correspondingly, w_m would be sufficiently below the Nash-bargained wage for the average productivity draw, $w\left[E\left(Z_2\right)\right]$). Assume instead that $\left[E\left\{Z\right\}^2 - Z^{m^2}\right] < \frac{\sigma^2}{4}$ and $\partial/\partial\sigma > 0$, as would be the case if $Z^m = E\left\{Z\right\}$. Then, a larger variance in Z leaves unchanged both the probability of outcomes where $Z < Z^m$ and profits equal zero, and the conditional expectation $E\{Z|Z < Z^m\} = 0$. On the contrary, the expected value of profits conditional on $Z > Z^m$ increases, since the probability of the event $Z > Z^m$ is unchanged, and equal to 1/2, while the average productivity outcome conditional on $Z > Z^m$ has increased. This in turn will raise the unconditional expectation of profits.

This example shows that there exist wage schedules such that expected profits may increase as uncertainty rises. For the case illustrated in the main text, an increase in uncertainty always lowers profits, as we prove in the following.

To find conditions for $\partial E_1(J_2)/\partial\sigma < 0$ we evaluate also the derivative of the first integral in equation (A-23). Evaluating the integral we obtain:

$$\begin{aligned}
&\int_{Z^a}^{Z^m} [y_2(Z_2) - w_m]\, dF(Z) \\
&= \int_{Z^a}^{Z^m} 2s\frac{1}{Z^b - Z^a}ds - \int_{Z^a}^{Z^m} w_m ds \\
&= \frac{1}{\sigma}\left[Z^{m^2} - \left(\overline{Z}^a - \frac{1}{2}\Delta\sigma\right)^2\right] + \\
&- \frac{1}{\sigma}w_m\left[Z^{m^2} - \left(\overline{Z}^a - \frac{1}{2}\Delta\sigma\right)^2\right] \\
&= \sigma^{-1}Z^{m^2} - \sigma^{-1}\overline{Z}^{a^2} - \frac{1}{4}\sigma - \frac{1}{4}\sigma^{-1}\overline{\sigma}^2 + \frac{1}{2}\overline{\sigma} \\
&+ \overline{Z}^a\overline{\sigma}\sigma^{-1} - w_m Z^m\sigma^{-1} + w_m\overline{Z}^a\sigma^{-1} - \frac{1}{2}w_m + \frac{1}{2}w_m\overline{\sigma}\sigma^{-1}
\end{aligned}$$

Evaluate the derivative of the latter expression:

$$\begin{aligned}
\partial/\partial\sigma &= -Z^{m^2}\sigma^{-2} + \overline{Z}^{a^2}\sigma^{-2} - \frac{1}{4} + \frac{1}{4}\overline{\sigma}^2\sigma^{-2} + \overline{Z}^a\overline{\sigma}\sigma^{-2} + w_m Z^m\sigma^{-2} - w_m\overline{Z}^a\sigma^{-2} - \frac{1}{2}w_m\overline{\sigma}\sigma^{-2} \\
&= \frac{1}{\sigma^2}\left[E\{Z\}^2 - Z^{m^2}\right] - \frac{1}{4} + w_m\frac{1}{\sigma^2}\left[Z_m - \overline{Z}^a - \frac{1}{2}\overline{\sigma}\right] \qquad \text{(A-26)}
\end{aligned}$$

Since the definition of Z^m and the properties of the uniform distribution imply that $w_m = Z^m$, we can combine equation (A-25) and (A-26) to obtain:

$$\begin{aligned}
\partial E_1(J_2)/\partial\sigma &= \frac{1}{2}\frac{1}{\sigma^2}\left[E\{Z\}^2 - Z^{m^2}\right] - Z^m\frac{1}{\sigma^2}\left[E\{Z\} - Z^m\right] - \frac{1}{8} \\
&= \left[E\{Z\} - Z^m\right] - \frac{\sigma}{2}.
\end{aligned}$$

Rearranging terms we obtain that the condition for $\partial E_1(J_2)/\partial\sigma$ to be less than zero can be written as:

$$\left[E\{Z\} - Z^m\right] < \frac{\sigma}{2}. \qquad \text{(A-27)}$$

Using equation (A-27) we obtain three results:

1. The condition (A-27) for $\partial E_1(J_2)/\partial\sigma$ to be less than zero can be rewritten as $Z^m > Z^a$, which is always verified since $Z^a = \min(Z)$. Therefore, an increase in uncertainty always lowers expected profits.

2. The absolute value of the derivative $\partial E_1(J_2)/\partial\sigma$ is increasing in σ. Therefore, for given Z^m the impact of an increase in uncertainty on expected profits is stronger, the larger the initial uncertainty in the distribution of Z, and the implied risk premium rises with uncertainty.

3. For values of $E\{Z\}$ such that $E\{Z\} > Z^m$, the absolute value of the derivative $\partial E_1(J_2)/\partial\sigma$ in-

creases as $Z^m \to E\{Z\}$. This implies that as the value of Z for which the wage-setting constraint becomes binding gets closer to the average productivity, the impact of a marginal increase in uncertainty becomes larger.■

In the context of our simple model, the last result of the proof describes the implications of increasing Z^m to a value closer to a given average productivity $E\{Z\}$. As Z^m increases, the probability of a draw $Z_2 < Z^m$ increases, and a given change in uncertainty has a larger impact on profits. However, the same result can be interpreted as describing how the impact of uncertainty on profits increases when lowering $E\{Z\}$ to a value closer to a given productivity limit Z^m. While this would be an unusual experiment in our simple model, it provides intuition on how the impact of an increase in uncertainty on profits is larger when output is below its average value, compared to the case when output is above its average value, in the context of the general equilibrium model presented in section 4. In this case, productivity is state-dependent, since it evolves according to an AR(1) process. Thus the average productivity draw depends on the past history of innovations, and $E\{Z_t\} = \rho Z_{t-1}$. In a recession, $[E\{Z_t\} - Z^m]$ is smaller than in an expansion for a given Z^m, and the probability of a draw $Z_t < Z^m$ increases. Thus, a given change in the variance of the innovation ε_{z_t} to the productivity process has a larger impact on profits, compared to the impact of the same change in the variance of ε_{z_t} in an expansion.[27]

Second-order stochastic dominance. An alternative way to prove that uncertainty lowers expected profits in our economy is to show that the distribution for $Z_{\overline{\sigma}}$ with range $\overline{\sigma}$ second-order stochastically dominates the distribution for Z_σ with range $\sigma > \overline{\sigma}$. Since the profit function, equal to $[y_2(Z_2) - w_m]$ for $Z \leq Z^m$ and to $\frac{y_2(Z_2)}{2}$ for $Z > Z^m$, is concave and weakly increasing, and $E(Z_{\overline{\sigma}}) = E(Z_\sigma)$, second-order stochastic dominance of $Z_{\overline{\sigma}}$ relative to Z_σ (and higher expected profits for the distribution $Z_{\overline{\sigma}}$ relative to Z_σ) can be proved by showing that $\int_{Z^a}^t G(Z)dZ \geq \int_{Z^a}^t F(Z)dZ \; \forall \; t > Z^a$ where $Z_\sigma \sim G(Z)$ and $Z_{\overline{\sigma}} \sim F(Z)$. We provide only the main steps of the proof, since the conditions for $\int_{Z^a}^t G(Z)dZ \geq \int_{Z^a}^t F(Z)dZ \; \forall \; t > Z^a$ are always satisfied when $G(Z)$ and $F(Z)$ intersect only once and $F(Z) < G(Z)$ for Z smaller than the productivity value corresponding to the intersection point, as in the case we are considering.

First, notice that the distribution functions $G(Z)$ and $F(Z)$ are weakly increasing, and that in the interval $[Z^a, \overline{Z}^a]$ it holds that $G(Z) > F(Z) = 0$. Then, since $G(E\{Z\}) = F(E\{Z\}) = 1/2$ we obtain that

$$G(Z) > F(Z) \; \forall \; Z \in [Z^a, E\{Z\})$$

This in turn implies:

$$\int_{Z^a}^t G(Z) > \int_{Z^a}^t F(Z)dZ \; \forall \; t \in [Z^a, E\{Z\}). \tag{A-28}$$

Since it holds that:

$$G(E\{Z\} - \varepsilon) - F(E\{Z\} - \varepsilon) = F(E\{Z\} + \varepsilon) - G(E\{Z\} + \varepsilon) \; \forall \; \varepsilon \geq 0$$

we obtain

$$\int_{E\{Z\}-Z^a}^0 G(E\{Z\} - \varepsilon) - F(E\{Z\} - \varepsilon)d\varepsilon + \int_0^{E\{Z\}-Z^a} G(E\{Z\} + \varepsilon) - F(E\{Z\} + \varepsilon)d\varepsilon = 0.$$

[27] Note that in the general equilibrium model, Z^m would be time-varying, since the state vector includes additional variables beside Z_t.

The previous result can be rewritten as

$$\int_{Z^a}^{E\{Z\}} G(Z) - F(Z)dZ + \int_{E\{Z\}}^{Z^b} G(Z) - F(Z)dZ$$
$$= \int_{Z^a}^{Z^b} G(Z) - F(Z)dZ = 0. \quad \text{(A-29)}$$

Since $G(Z)$ and $F(Z)$ are monotonically increasing, and also $\int_{Z^a}^{t} G(Z)dZ$ and $\int_{Z^a}^{t} F(Z)dZ$ are monotonically increasing, the results in equation (A-28) and (A-29) imply that $\int_{Z^a}^{t} G(Z)dZ \geq \int_{Z^a}^{t} F(Z)dZ$ $\forall\ t > Z^a$.■

The preceding proof highlights under what conditions an increase in variance may *increase* expected second-period profits. Under the assumptions in the main text, the profit function is concave, thus verifying that the random variable $Z_{\overline{\sigma}}$ second-order stochastically dominates the random variable Z_σ implies that the profit function is decreasing in the variance of Z. If the profit function is not concave in Z, the condition $\int_{Z^a}^{t} G(Z)dZ \geq \int_{Z^a}^{t} F(Z)dZ$ is not sufficient to ensure that $E_1(J_2|Z_{\overline{\sigma}}) > E_1(J_2|Z_\sigma)$, and there may exist parameterizations such that firms prefer higher variance in Z. One example is provided by a profit function equal to $\frac{y_2(Z_2)}{2}$ for $Z > Z^m$, and equal to zero for $Z \leq Z^m$, discussed above. In this case, an increase in the variance of Z will lower profits only if condition (A-25) is verified. It is straightforward to build constraints for wage setting resulting in non-concave profit functions, allowing for the possibility of a rise in profits as variance increases. For example, bargaining may be constrained by a higher value of w_{m_1} for $Z^{m_2} < Z < Z^{m_1}$, and a lower value $w_{m_2} < w_{m_1}$for $Z < Z^{m_2}$. Alternatively the firm may be constrained to pay a fixed w_{m_1} for $Z^{m_2} < Z < Z^{m_1}$, and pay the Nash wage $w = y(Z)/2$ for $Z < Z^{m_2}$, lowering the wage under the threat of bankruptcy.

A.2 Proof for Proposition 2

Part 1: When wage bargaining is constrained by the lower bound w_m, the total stream of expected profits falls when a mean-preserving spread of Z_2 lowers period 2 expected profits.

Pissarides (2009) shows that as long as the initial wage can be freely set, rigidity of wage payments for continuing matches is irrelevant for job creation. This result holds true in our simple model, only if wages can be freely set in period 1. We can rewrite the first-period Nash wage as:

$$w_1^{Nash} = \frac{y_1(Z_1)}{2} + \frac{1}{2}\left[E_1(J_2) - E_1(W_2)\right]$$
$$= Z + \frac{1}{2}\left[E_1(J_2) - E_1(W_2)\right] \quad \text{(A-30)}$$

where W_2 is the period-2 value of the match to the worker.

Note that $\frac{y(Z_1)}{2}$ is the wage that would be negotiated under unconstrained Nash bargaining in all period, when equal bargaining weight in period 2 implies $E_1(\Pi_2) - E_1(V_2) = 0$.

Assume that wages can be freely set in period 1, but are constrained by w_m in period 2. Whatever the level of the constraint, the period-2 profits are independent from the realization Z_1, and so is w_1^{Nash}. Since vacancy posting depends on the expected total profit from the match, we compute:

$$E_0(J_1) = \int_{Z^a}^{Z^b} \left[y_1(Z_1) - w_1^{Nash}\right] dF(Z) + E_0(J_2)$$

Using the wage solution, we obtain:

$$
\begin{aligned}
E_0(J_1) &= \int_{Z^a}^{Z^b} \left[y_1(Z_1) - \frac{y_1(Z_1)}{2} - \frac{1}{2}\left[E_1(J_2) - E_1(W_2)\right] \right] dF(Z) + E_0(J_2) \\
&= E(Z) - \frac{1}{2}\left[E_1(J_2) - E_1(W_2)\right] + E_0(J_2)
\end{aligned}
$$

where the term $E(Z)$ is the period-1 profit that would be obtained if Nash bargaining were unconstrained at all times. Observe then that $E_0(J_2)$ is a function of w_m, but the total surplus is not, given the fact that $t = 2$ is the terminal period. Then $E_1(J_2) + E_1(W_2) = G$ where G is a constant. This then implies that

$$
\begin{aligned}
E_0(J_1) &= E(Z) - \left[\frac{2E_0(J_2) - G}{2} \right] + E_0(J_2) \\
&= E(Z) + G/2
\end{aligned}
$$

which is constant. Therefore, the total profit from the match is independent from $E_0(J_2)$ and w_m, implying the wage bound at $t = 2$ leaves the ex-ante incentive to post vacancies unchanged.

Finally, note that this result only obtains because unconstrained Nash bargaining allowed for a *lower* wage - compared to the unconstrained case - in period 1, given that a lower wage limit w_m exists in period 2 and splits the surplus so that $E_1(J_2) - E_1(W_2) < 0$. If the wage could not be freely adjusted in period 1, $E_0(J_1)$ may fall whenever $E_0(J_2)$ falls and the wage cannot be lowered all the way to its Nash value defined in equation (A-30).

B Nash Bargaining

The real wage w_t maximizes the following objective function:

$$
J_t^{1-\eta} W_t^{\eta} - \Gamma_t.
$$

The first-order condition with respect to w_t implies:

$$
(1-\eta)\left(\frac{W_t}{J_t}\right)^{\eta} \frac{\partial J_t}{\partial w_t} + \eta \left(\frac{W_t}{J_t}\right)^{\eta-1} \frac{\partial W_t}{\partial w_t} - \Gamma_{w,t} = 0,
$$

where $\Gamma_{w,t} \equiv \partial\Gamma_t/\partial w_t$. Since $\partial J_t/\partial w_t = -\partial W_t/\partial w_t = -1$, we obtain:

$$
(\eta - 1)\left(\frac{W_t}{J_t}\right)^{\eta} + \eta \left(\frac{W_t}{J_t}\right)^{\eta-1} - \Gamma_{w,t} = 0. \tag{A-31}
$$

Equation (A-31) can be rearranged as follows:

$$
\eta J_t + (\eta - 1) W_t - \Gamma_{w,t} J_t^{\eta} W_t^{1-\eta} = 0.
$$

Let $\Lambda_t \equiv \Gamma_{w,t} J_t^{\eta} W_t^{1-\eta}$. Therefore:

$$
\eta J_t + (\eta - 1) W_t - \Lambda_t = 0. \tag{A-32}
$$

Using the first-order condition for vacancy posting, the firm's surplus can be written as

$$J_t = e^{Z_t} - w_t + \frac{\kappa}{q_t}. \tag{A-33}$$

Equation (16) implies:

$$W_t = w_t - b + (1-\lambda)(1-p_t) E_t (\beta_{t,t+1} W_{t+1}).$$

Using (A-32), we have

$$W_t = \frac{\eta J_t}{1-\eta} - \frac{\Lambda_t}{1-\eta},$$

which implies

$$W_t = w_t - b + (1-\lambda)(1-p_t) E_t \left[\beta_{t,t+1} \left(\frac{\eta J_{t+1}}{1-\eta} - \frac{\Lambda_{t+1}}{1-\eta} \right) \right],$$

or

$$W_t = w_t - b + \frac{\eta}{1-\eta}(1-p_t)\frac{\kappa}{q_t} - (1-\lambda)(1-p_t) E_t \left(\beta_{t,t+1} \frac{\Lambda_{t+1}}{1-\eta} \right). \tag{A-34}$$

Substituting (A-33) and (A-34) into the sharing rule, we finally obtain:

$$w_t = w_t^{flex} + \Omega_t,$$

where

$$w_t^{flex} \equiv \eta \left(e^{Z_t} + \kappa \frac{p_t}{q_t} \right) + (1-\eta) b$$

and

$$\Omega_t \equiv -\Lambda_t + (1-\lambda)(1-p_t) E_t [\beta_{t,t+1} \Lambda_{t+1}].$$

C Third-Order Policy Functions

The set of equilibrium conditions can be written as

$$0 = E_t [f (y_{t-1}, y_t, y_{t+1}, u_t; \delta)], \tag{A-35}$$

where the term f denotes a vector valued function, continuously n-times differentiable in all its arguments, y_t is the vector of endogenous variables, and u_t is the vector of exogenous, independently and identically distributed random variables. The auxiliary parameter $\delta \in [0,1]$ scales the uncertainty in the model: $\delta = 1$ corresponds to the stochastic model, while $\delta = 0$ represents the deterministic version of the model.

Consider a generic model solution of the form:

$$y_t = g (y_{s,t-1}, u_t; \delta), \tag{A-36}$$

where is the vector of state variables contained in y_t. Let $\bar{y}$ be the solution to (A-35) when $\delta = 0$ and $u_t = 0$ for any t. We refer to $\bar{y}$ as the deterministic steady state of the model. We approximate the unknown policy function $g(\cdot)$ by successively differentiating (A-35) around the deterministic steady state and solving the resulting system for the unknown policy coefficients. Below, we denote partial derivatives with subscripts.

A third-order Taylor expansion of (A-35) implies the following third-order approximation to the model policy functions:

$$
\begin{aligned}
y_t^{(3)} &= \bar{y} + g_y \left(y_{s,t-1}^{(3)} - \bar{y}_s \right) + g_u u_t \\
&+ \frac{1}{2} g_{yy} \left(y_{s,t-1}^{(3)} - \bar{y}_s \right)^{\otimes 2} + g_{yu} \left(y_{s,t-1}^{(3)} - \bar{y} \right) \otimes u_t + \frac{1}{2} g_{uu} \left(u_t \right)^{\otimes 2} + \frac{1}{2} g_{s2} + \\
&+ \frac{1}{6} g_{yyy} \left(y_{s,t-1}^{(3)} - \bar{y}_s \right)^{\otimes 3} + \frac{1}{6} g_{uuu} \left(u_t \right)^{\otimes 3} + \frac{1}{6} g_{s3} \\
&+ \frac{1}{2} g_{yyu} \left(y_{s,t-1}^{(3)} - \bar{y}_s \right)^{\otimes 2} \otimes u_t + \frac{1}{2} g_{yuu} \left(y_{s,t-1}^{(3)} - \bar{y} \right) \otimes \left(u_t \right)^{\otimes 2} + \\
&+ \frac{1}{2} g_{ys} \left(y_{s,t-1}^{(3)} - \bar{y}_s \right) + \frac{1}{2} g_{us} u_t .
\end{aligned} \tag{A-37}
$$

where the superscript (3) denotes the order of the approximation and $\bar{y}_s$ is the vector of states at the deterministic steady state.

Incremental Representation

Let $dy_t^{(1)} \equiv y_t^{(1)} - \bar{y}$ be the first-order increment, i.e., the variation in y_t that is captured by a first-order approximation to the model policy functions. This increment is given by

$$
dy_t^{(1)} = g_y dy_{s,t-1}^{(1)} + g_u u_t . \tag{A-38}
$$

Let the second-order increment be the difference between the first and second order approximation: $dy_t^{(2)} \equiv y_t^{(1)} - y_t^{(2)}$. It is easy to show that $dy_t^{(2)}$ is given by:

$$
dy_t^{(2)} = g_y dy_{s,t-1}^{(2)} + \frac{1}{2} g_{yy} \left(dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(2)} \right)^{\otimes 2} + g_{yu} \left(dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(2)} \right) \otimes u_t + \frac{1}{2} g_{uu} \left(u_t \right)^{\otimes 2} + \frac{1}{2} g_{s2}, \tag{A-39}
$$

where $\otimes$ denotes the Kronecker product. Finally, define the third order increment as the difference between the second and third order approximation: $dy_t^{(3)} \equiv y_t^{(3)} - y_t^{(2)}$. It is possible to show that

$$
\begin{aligned}
dy_t^{(3)} &= g_y dy_{s,t-1}^{(3)} + \frac{1}{2} g_{yy} \left[2 \left(dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(2)} \right) \otimes dy_{s,t-1}^{(3)} + \left(dy_{s,t-1}^{(3)} \right)^{\otimes 2} \right] + g_{yu} \left(dy_{s,t-1}^{(3)} \otimes u_t \right) \\
&+ \frac{1}{6} g_{yyy} \left(dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(2)} + dy_{s,t-1}^{(3)} \right)^{\otimes 3} + \frac{1}{6} g_{uuu} \left(u_t \right)^{\otimes 3} + \frac{1}{6} g_{s3} \\
&+ \frac{1}{2} g_{yyu} \left(dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(2)} + dy_{s,t-1}^{(3)} \right)^{\otimes 2} \otimes u_t + \frac{1}{2} g_{yuu} \left(dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(2)} + dy_{s,t-1}^{(3)} \right) \otimes \left(u_t \right)^{\otimes 2} \\
&+ \frac{1}{2} g_{ys} \left(dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(2)} + dy_{s,t-1}^{(3)} \right) + \frac{1}{2} g_{us} u_t .
\end{aligned} \tag{A-40}
$$

Then, using (A-38) to (A-40), the third-order accurate policy functions in (A-37) can be expressed as:

$$
y_t^{(3)} = \bar{y} + dy_t^{(1)} + dy_t^{(2)} + dy_t^{(3)} .
$$

Pruned Policy Functions

Let variables with a hat denote pruned solutions. The pruning algorithm proposed by Andreasen (2012) is defined by the following incremental recursion:

$$
d\tilde{y}_t = dy_t^{(1)} + d\tilde{y}_t^{(2)} + d\tilde{y}_t^{(3)},
$$

where:

$$dy_t^{(1)} = g_y dy_{s,t-1}^{(1)} + g_u u_t,$$

$$d\tilde{y}_t^{(2)} = g_y d\tilde{y}_{s,t-1}^{(2)} + \frac{1}{2} g_{yy} \left(dy_{s,t}^{(1)} \right)^{\otimes 2} + g_{yu} \left(dy_{s,t}^{(1)} \otimes u_t \right) + \frac{1}{2} g_{uu} \left(u_t \right)^{\otimes 2} + \frac{1}{2} g_{s2},$$

and

$$\begin{aligned} d\tilde{y}_t^{3} = {} & g_y d\tilde{y}_{s,t-1}^{(3)} + \frac{1}{2} g_{yy} d\tilde{y}_{s,t-1}^{(2)} \otimes \left[dy_{s,t-1}^{(1)} + dy_{s,t-1}^{(1)} \right] + g_{yu} d\tilde{y}_{s,t-1}^{(2)} \otimes u_t \\ & + \frac{1}{6} g_{yyy} \left(dy_{s,t-1}^{(1)} \right)^{\otimes 3} + \frac{1}{6} g_{uuu} \left(u_t \right)^{\otimes 3} + \frac{1}{6} g_{s3} \\ & + \frac{1}{2} g_{yyu} \left[\left(dy_{s,t-1}^{(1)} \right)^{\otimes 2} \otimes u_t \right] + \frac{1}{2} g_{yuu} \left[dy_{s,t-1}^{(1)} \otimes \left(u_t \right)^{\otimes 2} \right] \\ & + \frac{1}{2} g_{ys} dy_{s,t-1}^{(1)} + \frac{1}{2} g_{us} u_t. \end{aligned} \tag{A-41}$$

Intuitively, this algorithm chooses to keep both the quadratic and cubic term in the unpruned third-order approximation in (A-40). It prunes the quadratic term by replacing it with the Kronecker product of the first order approximation. The cubic term is replaced by the first-order approximation raising to the three-fold Kronecker power, and the Kronecker product of the pruned quadratic term and the first-order approximation.

D Impulse Response Functions (IRFs)

IRFs at the Ergodic Mean in the Absence of Shocks

First, we simulate the model for 1000 quarters without shocks starting from the non-stochastic steady state. We then check for convergence, i.e., whether the maximum change during the last 100 periods was bigger than $1e-12$. Starting at the ergodic mean in the absence of shocks, we compute the IRFs as the percentage difference of the respective variables between the system shocked with the respective shock and the baseline model response, i.e., the model response without shocks.

Generalized IRFs

As standard practice in the literature, we compute Generalized Impulse Response Functions (GIRFs) as follows: (i) draw a series of random shocks u_{zt} for $T = 300$ periods; (ii) simulate the model using the third-order policy functions and store the dynamics of the endogenous variables in the vector y_t; (iii) add one standard deviation to the simulated series of productivity shocks at period $t = 150$; (iv) simulate the model using the third-order policy functions and store the dynamics of the endogenous variables in the vector yy_t; (v) compute the IRFs for this experiment by constructing $yy_t - y_t$; (vi) repeat steps (i) to (v) for 5000 times and report the average. This "average" impulse response is the GIRFs. By construction, the GIRFs assumes that the economy is at the ergodic mean when the one standard deviation productivity shock is realized. Moreover, future productivity draws (after period t) are averaged out.

E Iterative Approximation Procedure to Obtain $\Gamma_{w,t}$

Here we describe the steps of the iterative procedure we use to parametrize the polynomial penalty function $\Gamma_{w,t}$.

First, we simulate the model with unconstrained Nash wage bargaining to obtain the unconstrained ergodic wage distribution. We use the third-order, pruned policy functions, and set the volatility of the exogenous productivity process, σ_z, to match the (HP-filtered) absolute standard deviation of U.S. aggregate output for the period 1972Q1 to 2010Q4. (As is common practice in the literature, we set the smoothing parameter equal to 1600. We average $N = 1000$ simulations, each of length $T = 250$ periods.) By pooling all the simulated wage paths, we obtain the ergodic wage distribution in the unconstrained model. We use this distribution to determine the frequency of wage outcomes eliminated by in the model that features the OBC.

Next, we perform the following iterative procedure:

1. Choose a candidate function $\Gamma_{w,t}$ (see below).

2. Choose σ_z.

3. Simulate the model with the OBC to obtain the constrained ergodic wage distribution. As before, consider $N = 1000$ and $T = 250$.

4. Verify that the following criteria are met:

   (a) The HP-filtered standard deviation of output matches the data.

   (b) The lowest wage value observed in the ergodic distribution corresponds to the 26^{th} percentile of the unconstrained ergodic wage distribution.

   (c) $Max(\Gamma_{w,t}) < 2\%$ for or any $w_t \in \left[w, w^{95th}\right]$, where w is the steady-state wage and w^{95th} is the wage corresponding to 95^{th} percentile in the ergodic wage distribution for the model with unconstrained wage bargaining.

5. Repeat (1) to (4) until the criteria are met.

E.0.1 ***Choice of the Candidate Function***

Least Square Approximation Assume ordinary polynomials as the basis functions $\Gamma_i(w_t)$ and a uniform grid for the approximation points. We define the basis functions such that

$$\Gamma_{w,t} = \alpha_1 \left(\widetilde{w}_t - \widetilde{w}\right) + \alpha_2 \left(\widetilde{w}_t - \widetilde{w}\right)^2 + \alpha_3 \left(\widetilde{w}_t - \widetilde{w}\right)^3 + \omega, \tag{A-42}$$

where $\widetilde{w}$ is a given constant and $\widetilde{w}_t \equiv w_t + (\widetilde{w} - w)$. Thus, $\widetilde{w}_t$ is equal to $\widetilde{w}$ in the deterministic steady state but it is identical to w_t in log-deviations. We set ω such that $\Gamma_{w,t}$ is 0 in the deterministic steady state, i.e., when $w_t = w$. Notice that an alternative candidate function would be

$$\Gamma_{w,t} = \alpha_1 \left(w_t - w\right) + \alpha_2 \left(w_t - w\right)^2 + \alpha_3 \left(w_t - w\right)^3,$$

such that $\Gamma_{w,t} = 0$ whenever $w_t = w$, both in the deterministic steady state and in response to aggregate shocks.

To complete step 1, we need to specify a grid of values $X_t = \left[\left(\tilde{w}_0 - \tilde{w}\right), .., \left(\tilde{w}_N - \tilde{w}\right)\right]$ and a corresponding set of values $Y_t = f\left(X_t\right)$. Then we estimate the coefficients in equation (A-42) by regressing Y_t on X_t using a non-linear least squares algorithm. We determine the values in Y_t by using the following target function:

$$\Gamma^L_{w,t} \equiv \frac{\phi}{\psi}\left[1 - e^{-\psi(w_t - w)}\right], \tag{A-43}$$

although the least squares algorithm does not require the target function to be neither continuous nor differentiable. Our choice reflects the fact that the approximation problem is better conditioned when choosing a smooth target function. The iteration converged for values $\phi = 7$ and $\psi = 380$ to a polynomial such that the value of the penalty is equal to 40 percent of the steady-state wage for the lowest outcome of w_t, which endogenously determines w_m. Given our parameterization, w_m is approximately 1 percent below the steady-state wage. While this value may appear very close to the steady state, recall that it meets our requirement that w_m corresponds to the 26^{th} percentile of the unconstrained Nash bargaining ergodic distribution. The reason is that in our model wage dispersion is smaller than in U.S. data. This in turn is the consequence of choosing a parameterization for the other coefficients in the model able to generate a sufficient volatility in employment even under unconstrained Nash bargaining.

Chebyshev Polynomials Interpolation Alternatively, we could assume Chebyshev polynomial as the basis functions $\Gamma_i(w_t)$, and the Chebyshev nodes as the interpolation points over an interval $[w_{\min}, w_{\max}]$. The candidate polynomial can be found by interpolating a target function, such as the function defined in equation (A-43). We experimented with this approach, and found that, as the least squares method, it is better conditioned when using a smooth function as target.

F Accuracy

Euler Equation Error Test

We assess the accuracy of the approximated solution using the bounded rationality metric in Judd (1998). The Euler equation residuals quantify the error in the intertemporal allocation problem using units of consumption.

To begin, recall the Euler equation for job creation:

$$\frac{\kappa}{q_t} = \beta(1-\lambda)E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[Z_{t+1} - w_{t+1} + \frac{\kappa}{q_{t+1}}\right]\right\}.$$

Since the solution method is an approximation, the above equation will not hold exactly when evaluated using the computed decision rules. For this reason, we can define the Euler equation error (expressed as a fraction of units of consumption) as

$$err_t = \frac{-C_t + \left\{\beta(1-\lambda)\frac{q_t}{k}E_t\left[C_{t+1}^{-\gamma}\left(\frac{\kappa}{q_{t+1}} + Z_{t+1} - w_{t+1}\right)\right]\right\}^{-\frac{1}{\gamma}}}{C_t}. \tag{A-44}$$

We compute Euler errors for different initial levels of productivity. Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) suggest to simulate the model density to identify a plausible range of the state space in which the accuracy test should be conducted. However, since we are interested in evaluating the accuracy of the unpruned policy functions, we cannot immediately apply such procedure because of the explosive behavior of model simulations. As a consequence, we consider productivity shocks from a range 0.015 to 0.015, the same range considered in the paper. We evaluate the conditional expectation by simulating $20,000$ realizations using the state-space system.

We summarize the information from Euler equation error functions by reporting the maximum Euler error (expressed in logarithmic scale with base 10). The maximum Euler error is useful as a measure of accuracy because it bounds the mistake that we are incurring owing to the approximation. Also, the literature on numerical analysis has found that maximum errors are good predictors of the overall performance of a solution. The maximum error across the alternative initial productivity values we consider is -2.87, which is not too distant from the values reported by Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) for a benchmark real business cycle model approximated with a fifth-order Taylor expansion (see Table 5 on page 2052 of their paper).

F.1 Comparison to Taylor Expansion of Penalty Function

In this section, we contrast our approximation method to the standard approach used in the literature. In our context, the standard approach amounts to specifying a continuos, twice-differentiable penalty function, approximating its first derivative with a third-order Taylor expansion. We choose as penalty function the Linex function:

$$\Gamma_t^L = \frac{\phi}{\psi^2}\left[e^{-\psi(w_t - w)} + \psi(w_t - w) - 1\right],$$

whose first-derivative is defied by equation (A-43). The Linex has the advantage of being an entire function: As the order of the approximation increases, the Taylor approximation of $\Gamma_{w,t}^L$ converges to $\Gamma_{w,t}^L$ over the whole domain. We show that up to the third order, the Taylor expansion diverges substantially from $\Gamma_{w,t}^L$ in regions of the state space that are of economic interest. By contrast, in the neighborhood of the deterministic steady state, the model dynamics implied by our methodology reproduce closely those obtained with a third-order Taylor expansion of the function in (A-43).

Figure A.1 compares the polynomial penalty function $\Gamma_{w,t}$ in (21) to a third-order approximation of $\Gamma_{w,t}^L$. We inspect the behavior of these two alternative penalty function as a function of the wage w_t. In Figure A.1, the solid line plots $\Gamma_{w,t}^L$, the dotted line plots its third-order Taylor expansion, and the dashed line plots the polynomial penalty function $\Gamma_{w,t}$ in (21) implied by the least square estimation. To facilitate the comparison, the plots are scaled so that $\widetilde{w}_m$ is equal to the steady-state wage. Therefore, the x-axis plots the wage as a fraction of the steady-state wage, normalized to 1. The y-axis reports the value of the first derivative of the penalty function as a fraction of the steady state wage.

Fig A.2 presents the ergodic wage distribution implied by the Taylor approximation of $\Gamma_{w,t}^L$. The distribution is truncated for both high and low values of the wage. In effect, the Taylor expansion implies that the solution behaves as if approximating two occasionally binding constraints, when only one exists in the model. Since the Taylor expansion of $\Gamma_{w,t}^L$ changes sign symmetrically around w_m, it can be seen from equation (20) that an increase in w_t above the wage floor results in a positive value for Λ_t, leading, *ceteris paribus*, to a rise in w_t smaller than the rise in the Nash-efficient wage $\widetilde{w}_t^{nash}$ for a given TFP shock. In equilibrium this reduces the value of the surplus, constraining the realization of w_t from above for sufficiently high values of TFP.

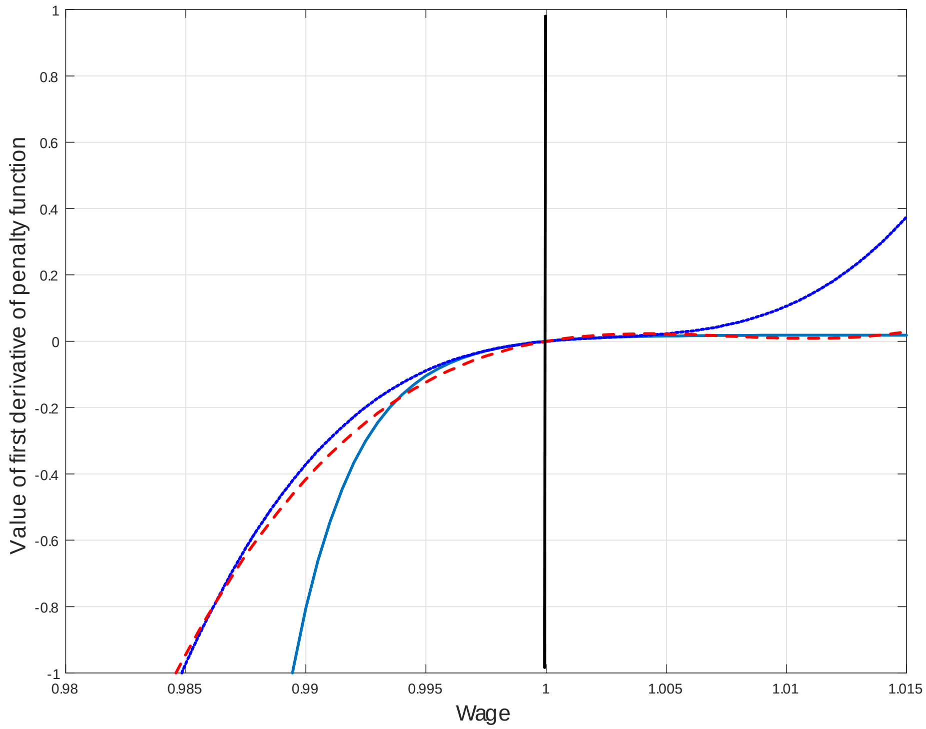

Figure A.1. Derivative of a Linex function with parameters $\phi = 7$ and $\psi = 380$ (*solid line*); Third-order Taylor approximation of the Linex derivative (*dotted line*); Polynomial approximation (*dashed line*). The wage is normalized to 1 at the approximation point. The value of the first derivative of the penalty function is scaled as a share of the steady-state wage.

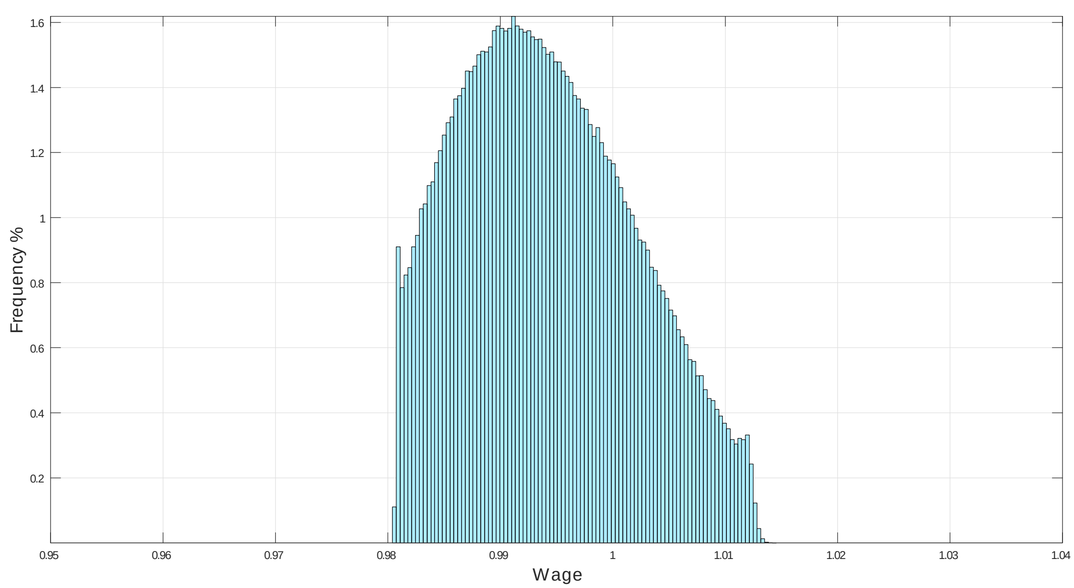

Figure A.2. Ergodic wage distribution, third-order Taylor approximation of the derivative of a Linex function with parameters $\phi = 7$ and $\psi = 380$.